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THESIS

MILLIMETER WAVE FILTER DESIGN WITH SUSPENDED STRIPLINE

by

Anastasios K. Kotronis

September 1989

Thesis Advisor Harry A. Atwater

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The scope of this thesis is to establish design rules for the shielded form of Suspended Substrate Stripline (SSL) as a propagation medium, and also to introduce a practical model for calculation of the gap dimensions of the SSL.

The results of this study show that there are available accurate and simple design formulas for the analysis and synthesis calculations of SSL transmission line parameters. It is also shown that the proposed method for analysis of the gap discontinuity can be used for practical applications with accurate results.

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Millimeter-Wave Filter Design With Suspended Stripline

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

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September 1989

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ABSTRACT

The scope of this thesis is to establish design rules for the shielded form of Suspended Substrate Stripline (SSL) as a propagation medium, and also to introduce a practical model for calculation of the gap dimensions of the SSL.

The results of this study show that there are available accurate and simple design formulas for the analysis and synthesis calculations of SSL transmission line parameters. It is also shown that the proposed method for analysis of the gap discontinuity can be used for practical applications with accurate results.

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I. INTRODUCTION

Planar and integrated microwave circuitry for application in radar and guidance systems is predominantly based on the microstrip transmission medium. Due to high microstrip losses with increasing frequency, lower-loss media are required for frequencies at K-band (30GHz) and above. These higher frequencies are coming into increased use for higher target resolution and improved security.

The important alternative microwave transmission media which preserve the advantages of planar printed circuit forms are the finline and shielded suspended-substrate systems. Of those two media, the finline has had extensive application and theoretical investigation. The shielded suspended-substrate stripline (SSL) has the potential for very low transmission loss since, in its quasi-TEM propagation mode, the current densities are very low due to its additional air gap, relative to microstrip. The SSL is at present a developmental system, and well-verified design rules are not yet available for it.

A number of empirical formulas for the computation of the SSL transmission line parameters have been published in the literature, but the validity and accuracy of these formulas is not known. At present, essentially no information is available for discontinuities such as capacitive gaps or angles in this medium. The series capacitive gap is of particular interest in that a useful microwave filter can be built using resonator sections coupled by series capacitive gaps. Keywords: microwave circuits

Of the candidate theoretical tools available for the computation of electromagnetic field problems: the method of moments (Galerkin method), and the variational-solution method. The variational method constitutes the most suitable approach for obtaining the wanted theoretical confirmation of SSL TEM-mode transmission parameters. Only a limited amount of preliminary analytical reduction is required for this method and well-developed variational formulas are available in the literature for the line geometry and the discontinuities of interest.

The scope of this thesis is to compare the available methods for analysis of SSL transmission line parameters and to select a suitable approach for practical computations, and also to examine and implement a practical model for calculation of the gap capacitance in the SSL.

The verification of the SSL transmission formulas which is the objective of this research will facilitate the application of this low loss transmission medium in radar and

millimeter wave system design and contribute to the available knowledge of the factors related to the practical utilization of the system.

The thesis begins in Chapter 2 with the examination of available models for the analysis of transmission line parameters of shielded SSL. After a comparison between the models, the most suitable model for implementation is proposed.

In Chapter 3 the calculation of static capacitance of a series gap in shielded SSL is presented, based on a theoretical method described in [Ref. 4].

In Chapter 4 the final SSL-filter design is shown, based on the basic filter theory and on the results from Chapters 2 and 3. In this chapter also, the designed filter is evaluated by using the Touchstone CAD Program available in the Microwave Laboratory of Naval Postgraduate School.

Chapter 5 includes the conclusions and recommendations of this study.

II. MODELS FOR ANALYSIS OF SSL TRANSMISSION LINE PARAMETERS

The objective of this chapter is to examine published closed-form design formulas for the Suspended Stripline (SSL) parameters, and to select accurate and practical expressions for implementation.

Attention in this work is restricted to TEM-mode propagation parameters. This limitation excludes the dispersive (frequency-dependent) effects which result from hybrid-mode wave propagation, which occurs in mixed-dielectric media. This simplifies the analytical task, and experience has shown that the TEM model is satisfactory for microwave frequencies up to 10 Ghz and is frequently extended to higher frequencies.

In section A two published closed-form design models for shielded SSL are examined, which introduce design formulas for the shielded transmission line parameters. The dominant interest in this investigation is the shielded form of suspended stripline typically operated within a standard waveguide as the shielding element. The advantages of shielded operation for microwave transmissions are that it leads to reduced radiation and limited interaction with neighboring components. In section B two available fundamental methods for **analysis** of SSL are examined which are based on electromagnetic field theory. These are used to test the formulas cited in part A. In section C, the published empirical **design** formulas for unshielded SSL are tested and the results are compared with the formulas for shielded SSL. Finally, in section D, the most suitable formulas for the foregoing procedures are proposed for the computation of SSL transmission line parameters.

A. CLOSED-FORM DESIGN EQUATIONS

1. Analysis Calculations

The single available published source of closed-form expressions for shielded suspended stripline is that of Ref. 1, p. 693.

In the reference closed-form analysis equations for shielded SSL are developed, by using least-square curve fitting to numerical results of the finite-difference method.

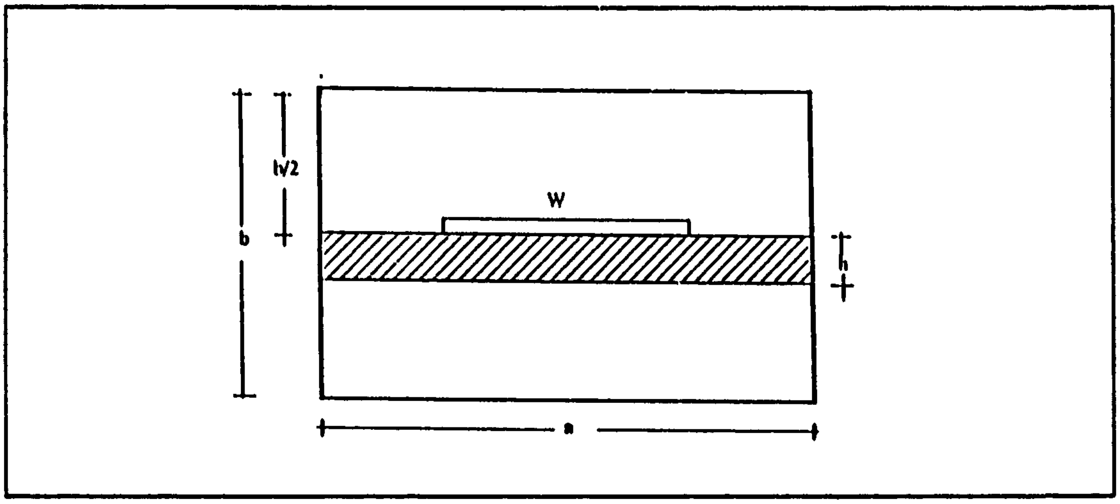

Figure 1. Cross-Sectional View of the Transmission Line to be Analyzed.

The design equations of Ref. 1 are shown below:
The characteristic impedance can be expressed as

$$Z = \frac{Z_0}{\sqrt{\varepsilon_e}} \tag{1}$$

where:

- Z_0 is the characteristic impedance of SSL of identical dimensions and completely filled with air, and
- ε_e is the effective dielectric constant of SSL.

The effective dielectric constant is given

$$\sqrt{\varepsilon_e} = [1 + (E - F \ln \frac{W}{b}) \ln \frac{1}{\sqrt{\varepsilon_r}}]^{-1} \tag{2}$$

where, for $0 < W < a/2$,

$$E = 0.2077 + 1.2177(\frac{h}{b}) - 0.08364(\frac{a}{b}) \tag{3}$$

$$F = 0.03451 - 0.1031(\frac{h}{b}) + 0.01742(\frac{a}{b}) \tag{4}$$

or, for a/2 < w < a,

$$E = 0.4640 + 0.9647(\frac{h}{b}) - 0.2063(\frac{a}{b}) \tag{5}$$

$$F = -0.1424 + 0.3017(\frac{h}{b}) - 0.02411(\frac{a}{b}) \tag{6}$$

The characteristic impedance is given :

- for 0 < W < a/2

$$\sqrt{\varepsilon_e}\, Z_0 = \frac{\eta_0}{2\pi}[V + R\ln(\frac{6b}{W} + \sqrt{1 + \frac{4}{(W/b)^2}})] \tag{7}$$

where

$$\eta_0 = 120\pi \tag{8}$$

$$V = -1.7866 - 0.2035(\frac{h}{b}) + 0.4750(\frac{a}{b}) \tag{9}$$

$$R = 1.0835 + 0.1007(\frac{h}{b}) - 0.09457(\frac{a}{b}) \tag{10}$$

- for a/2 < W < a

$$\sqrt{\varepsilon_e}\, Z_0 = \eta_0[V + R[\frac{W}{b} + 1.3930 + 0.6670\ln(\frac{W}{b} + 1.444)]^{-1}] \tag{11}$$

where

$$V = -0.6301 - 0.07082(\frac{h}{b}) + 0.2470(\frac{a}{b}) \tag{12}$$

$$R = 1.9492 + 0.1553(\frac{h}{b}) - 0.5123(\frac{a}{b}) \tag{13}$$

where W, h, a, b are as shown in Fig. 1.

The range of structural and substrate constants, over which the developed equations are valid is:

1 < a/b < 2.5

1 < ε_r < 4

0.1 < h/b < 0.5

2. Synthesis Calculations

In this work the term **synthesis** is used to denote the computation of transmission system dimensions [Ref. 2: p. 331] to achieve specified transmission parameters, Z and ε_r, in keeping with the convention established in transmission design. The computation of parameters of a line of given dimensions is called the analysis process.

In the method of Ref. 2, a set of simple and explicit synthesis equations for SSL was developed using least-square curve fitting to numerical results obtained using Super Compact for SSL.

For specified characteristic impedance Z of SSL, the strip width w can be synthesized using the following formula

$$\frac{W}{b} = [A\exp(1.77245\frac{CZ}{120}) + B\exp(-1.77245\frac{DZ}{120})]^{-1} \qquad (14)$$

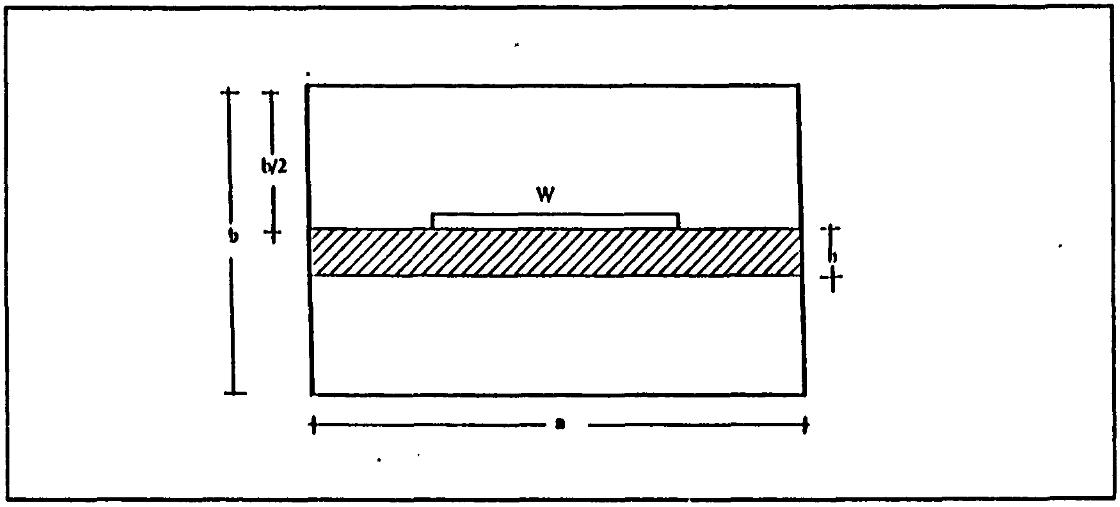

Figure 2. Cross Section of SSL.

where A, B, C, D have different values corresponding to the dielectric constants of the substrate as follows:

1. For $\varepsilon_r = 2.22$

$$A = 0.0854(\frac{a}{b})^{-3} + 0.2901(\frac{h}{b}) + 0.2342 - (\frac{h}{2b})^3\delta(\frac{h}{b} - 0.4) \qquad (15)$$

where

$$\delta(\frac{h}{b} - 0.4) = 1 \quad for\ h/b \geq 0.4 \qquad (16)$$

$$= 0 \quad \text{for } h/b < 0.4$$

$$B = -1.20831 \ln(\frac{a}{b}) - 0.3796 \ln(\frac{h}{b}) + 0.1370 \quad (17)$$

C = 1.446
D = 2.010

2. For $\varepsilon_r = 2.80$

$$A = [0.09680 \ln(\frac{h}{b}) + 0.6130]/[0.3980 \ln(\frac{a}{b}) + 1.0276] \quad (18)$$

$$B = 1.038(\frac{h}{b}) - 0.754(\frac{a}{b}) - 0.7380 \ln(\frac{h}{b}) - 0.4723 \quad (19)$$

C = 1.440
D = 2.005

3. For $\varepsilon_r = 3.80$

$$A = [0.1138 \ln(\frac{h}{b}) + 0.5775]/[0.4902 \log(\frac{a}{b}) + 1.0238] \quad (20)$$

$$B = -1.6943 \ln(\frac{h}{b}) - 1.544 - (\frac{a}{b} - 1)/(3.5 \frac{h}{b}) \quad (21)$$

C = 1.685
D = 2.792

The synthesis equations of SSL are valid over under following conditions:

$$40\Omega \leq Z \leq 150\Omega \quad (22)$$

$$1 \leq \frac{a}{b} \leq 2.5 \quad (23)$$

$$0.1 \leq \frac{h}{b} \leq 0.5 \quad (24)$$

3. Implementation

The closed-form equations of the sections above were evaluated in WR-28 waveguide with $\varepsilon_{eff} = 2.22$ and substrate height h = 0.5 mm.

Figure. 3 shows the resulting calculations of SSL characteristic impedance Z_0, for stripwidths W from 1 to 7 mm.

The discontinuity in the "Analysis curve" at W = A/2 occurs at the transition between the two sets of design equations for the ranges of $0 < W < \frac{A}{2}$ and $\frac{A}{2} < W < A$.

As is shown in the same figure the agreement between the Analysis and Synthesis calculations is within 1% for the whole range of W.

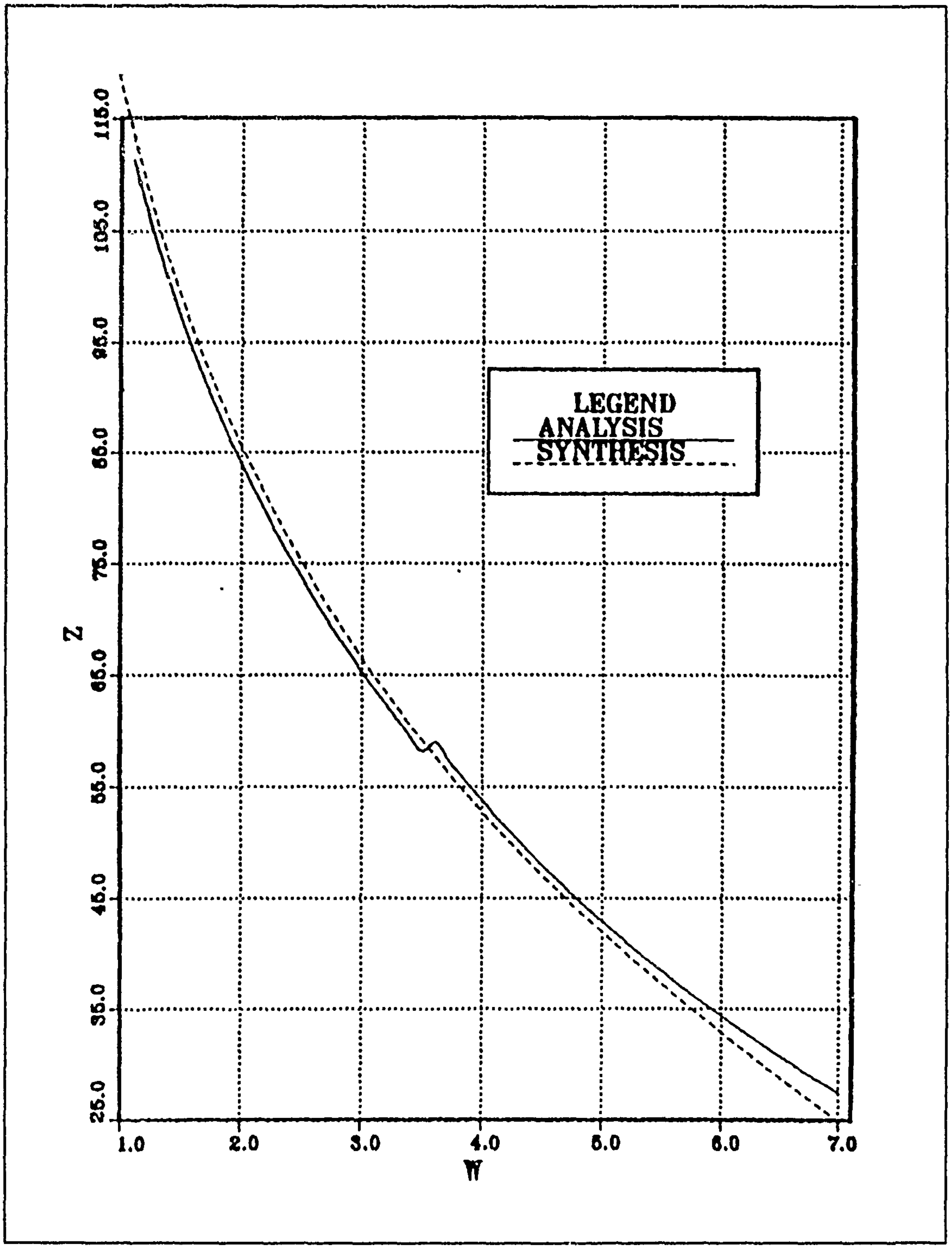

Figure 3. Comparison between Analysis and Synthesis Calculations.

B. ELECTROMAGNETIC-FIELD MODELS FOR SSL

The theoretical models used in this comparison were a variational calculation based on Ref. 3, p. 238 and a Quasi-static solution of Laplace's equation based on Ref. 4 with boundary-point matching over a number of points on the dielectric surface containing the strip conductor.

The variational method uses Green's functions for formulating the problem and a variational principle for obtaining line capacitances.

The first method assumes the charge density across the strip conductor as a trial function with maxima at the edges of the strip, while the second method specifies only the boundary potentials, a consideration which simplifies the problem.

1. Variational Calculation Method

In this method a various trial surface charge distributions were used for the charge on the conducting strip [Ref. 3: p. 238].

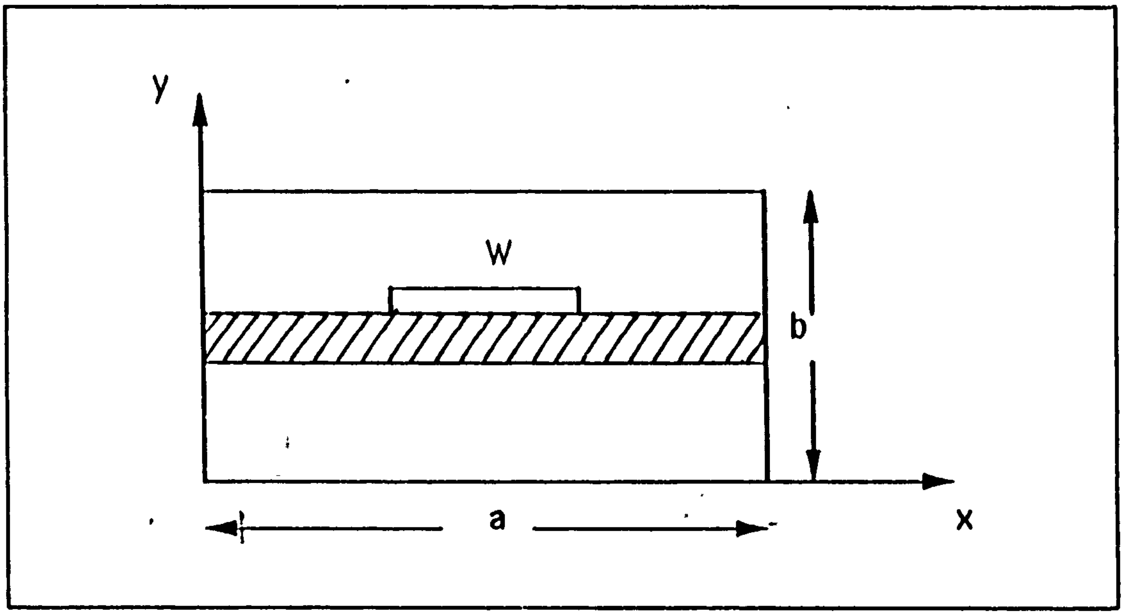

Figure 4. Cross-Sectional View of the Transmission Line to be Analyzed.

The static capacitance per unit length of a TEM transmission-system configuration can be obtained from a calculation of the electrostatic field energy W_E and with use of the expression:

$$W_E = \frac{Q^2}{2C} \tag{25}$$

where Q is the charge per electrode and C the unknown capacitance.

The energy W_E can be computed from

$$W_E = \iiint \rho \Phi dv \tag{26}$$

where Φ is the electrostatic potential and ρ is the charge density on the strip conductor.

With the z-coordinate taken along the axis on the strip transmission line the potential Φ in the transverse plane can be obtained from the charge distribution by means of a Green's function

$$\Phi(x,y) = \iint \rho(x',y')G(x,y;x',y')dx'dy' \tag{27}$$

Suitable forms of Green's function for the SSL geometry are available in the literature [Ref. 3: p. 239], and for reference are given in the Appendix B.

The charge Q in Equation (25) is calculated from

$$Q = \iint \rho(x,y)dxdy \tag{28}$$

where ρ is the charge distribution on the strip conductor. This distribution is not known a priori, but for the strip conductor is known to be concentrated at both edges, relative to that at the center.

For the charge density the following form of the trial function is assumed:

$$f(x) = \frac{1}{W}\left[1 + k\left|\frac{2}{W}\left(x - \frac{a}{2}\right)\right|^3\right] \quad for \quad \frac{a}{2} - \frac{W}{2} \leq x \leq \frac{a}{2} + \frac{W}{2} \qquad (29)$$

$$= 0 \qquad\qquad \text{otherwise}$$

where k is a constant to determine the shape of f(x). This constant can be chosen such as to maximize the line capacitance C, in the sense of the variational calculation.

With the capacitance C per unit length of the line available, the transmission-line parameters then may be calculated as follows

Characteristic impedance

$$Z = \frac{1}{3x10^8\sqrt{CC_0}} \qquad (30)$$

Transmission line wavelength at frequency f:

$$\lambda = \frac{3x10^8}{f}\sqrt{\frac{C_0}{C}} \qquad (31)$$

where C_0 is the value of capacitance when the dielectric layer is replaced by air. The complete calculation from [Ref. 3] is given in Appendix A, for reference.

Equations (30) and (31) were calculated for WR-28 waveguide, and Fig. 6 shows the resulting calculations of the SSL impedance Z_0 for $\varepsilon_r = 2.22$, substrate height h=0.5 mm and linewidth W between 1 and 7 mm.

2. Quasi-static Solutions of Laplace's Equation

The necessity for constructing a postulated charge distribution, as required for the variational method, can be avoided by treating the capacitance calculation as a standard boundary-value problem in Laplace's equation.

With this approach [Ref. 4] , a series solution is written for each of the three regions in the line cross section as in the figure below.

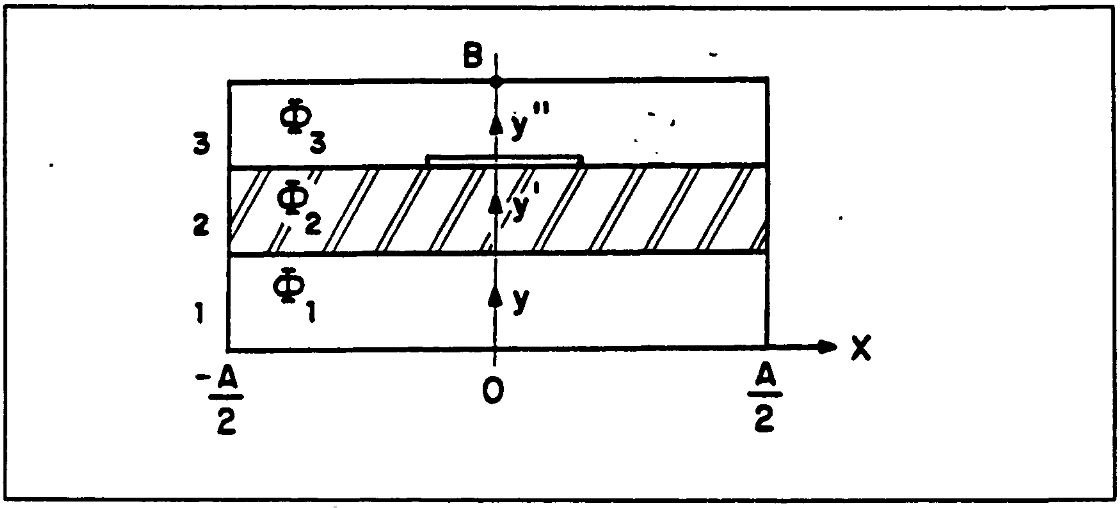

Figure 5. Cross Section of SSL.

If a potential of 1 Volt is assigned to the strip conductor, with 0 Volts on the outer shield boundary, then the capacitance can be obtained as the integral of the normal D-vector over a bounding surface enclosing the center conductor where

$$D = -\varepsilon \nabla \phi \tag{32}$$

The assumed solution in region 3 is

$$\Phi_3 = \sum_{n}^{N} D_n \sinh(k_n(y'' - h_3)) \cos(k_n x) \tag{33}$$

where the coefficients D_n are evaluated at N selected points over the surfase containing the strip conductor. The number of boundary matching points N, determines the accuracy of the solution and the size of the matrix to be solved.

Solutions of the type of equation (33) also hold in regions (1) and (2). The coefficients of these solutions are evaluated by enforcing boundary conditions at the interfaces between regions. The design equations from [Ref. 4] are given in Appendix B for reference. These equations were tested in WR-28 waveguide and Fig. 7 shows the resulting calculations of the SSL impedance Z_0 for $\varepsilon_r = 2.22$, h = 0.5 mm and W between 1 and 7 mm.

3. Comparison Between Closed Form and E/M Field Model Calculations

The Analysis and Synthesis closed-form equations, described in part A, were tested against the more fundamental calculations based on the electromagnetic field models, described in part B.

Figure 8 shows the resulting comparison between those four calculations of the SSL impedance Z_0, for $\varepsilon_r = 2.22$, substrate thickness h = 0.5 mm and stripwidth W between 1 and 7 mm. As shown in Fig. 8, the agreement is within 1% for W from 1 and 3 mm, and 8% or better for W from 3 to 7, with the greater accuracy at the more frequently used line widths.

C. COMPARISON OF DESIGN EQUATIONS FOR SHIELDED AND UNSHIELDED SSL.

Although numerous closed-form expressions exist for the unshielded form of SSL [Ref. 5: p. 173], [Ref. 6: p. 453], [Ref. 7: p. 1429], it was found by comparative calculation of the two cases (shielded and unshielded) that the presence of shielding has a major effect on the propagation characteristics of the line and that the line parameters may differ by 25% to 40% between shielded and unshielded lines of similar dimensions. Figure 9 shows the resulting comparison.

Therefore the unshielded-line design equations are not valid for the case of Shielded SSL.

D. CONCLUSION.

In view of the results reported above we can conclude that:

- The analysis formulae [Ref. 1: p. 693] and the synthesis formulae [Ref. 2: p. 331] introduce accurate and simple relations for calculation of SSL transmission line parameters.
- The formulas for the ushielded SSL are non-valid for use in the shielded SSL calculations.

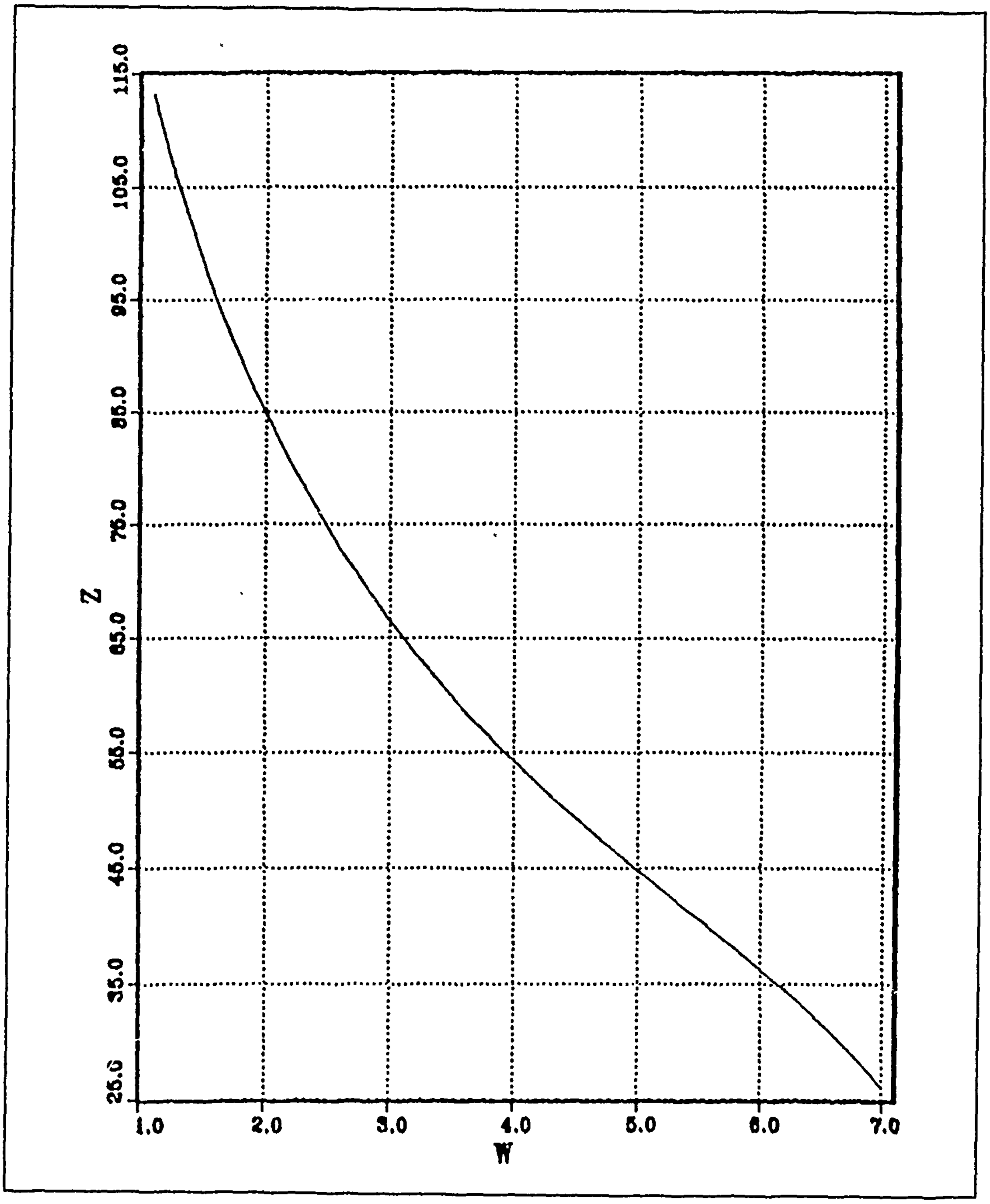

Figure 6. Characteristic Impedance Estimated Using the Variational Method.

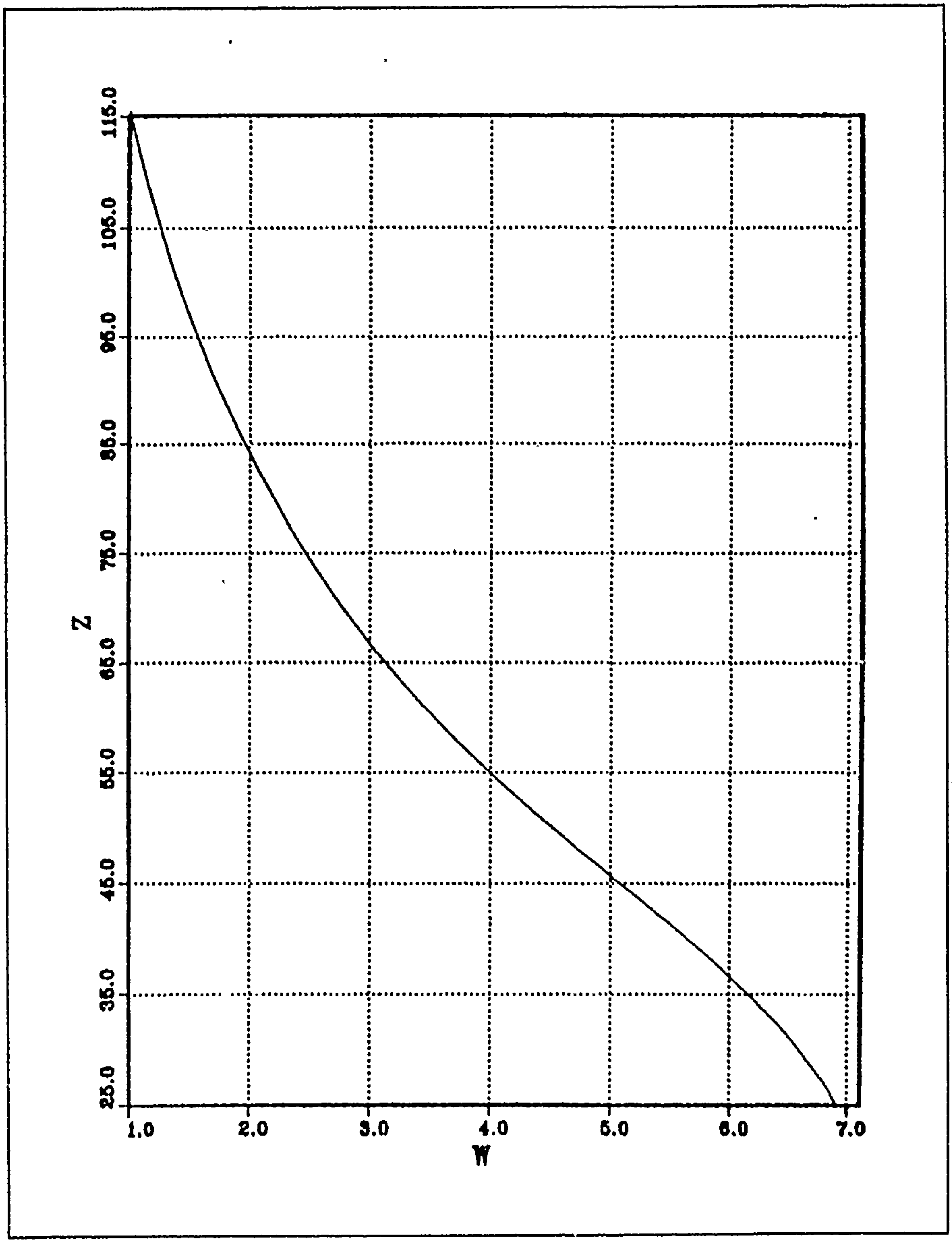

Figure 7. Characteristic Impedance Estimated Using the Quasistatic Method.

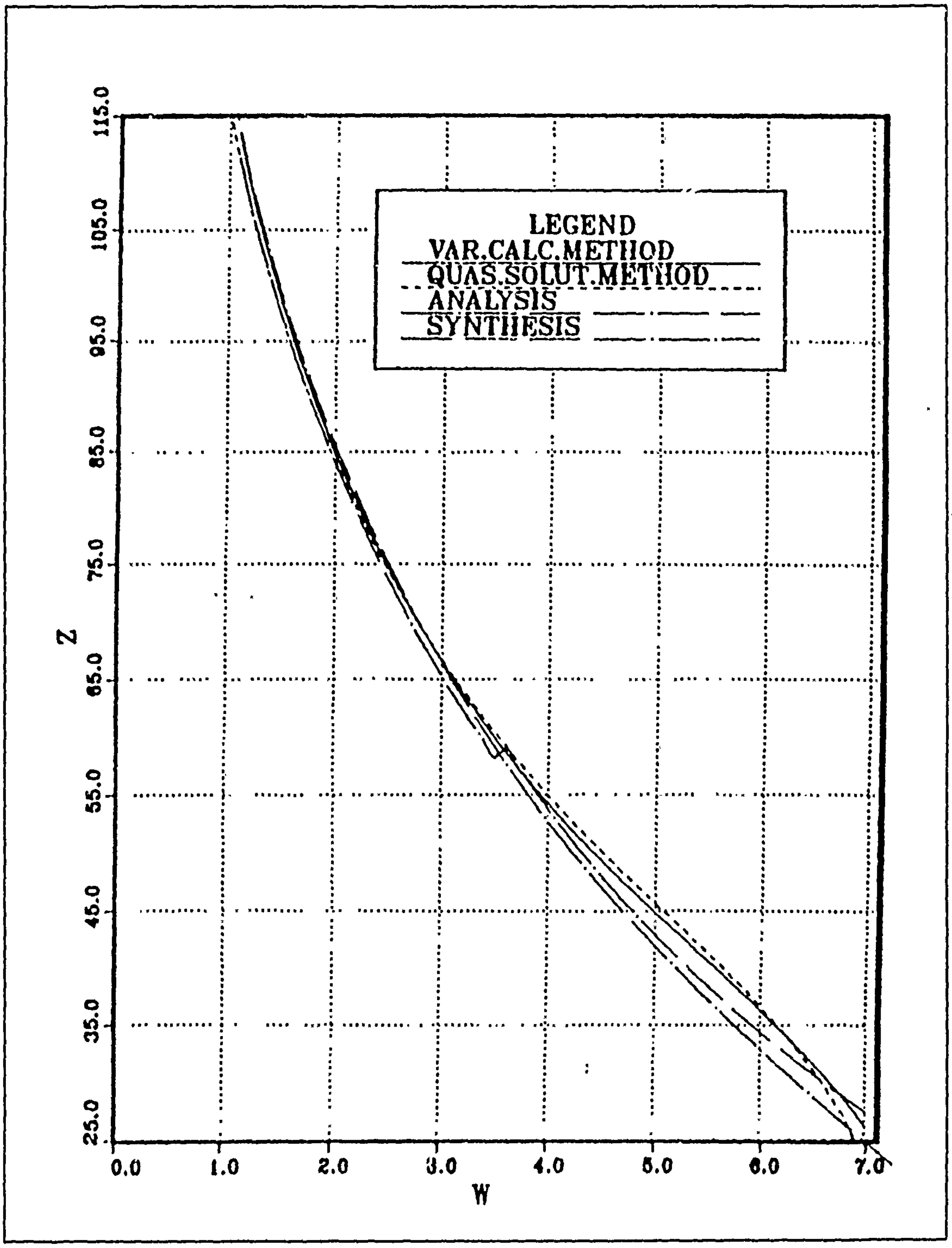

Figure 8. Comparison Between Closed-Form and E/M Field Model Calculations.

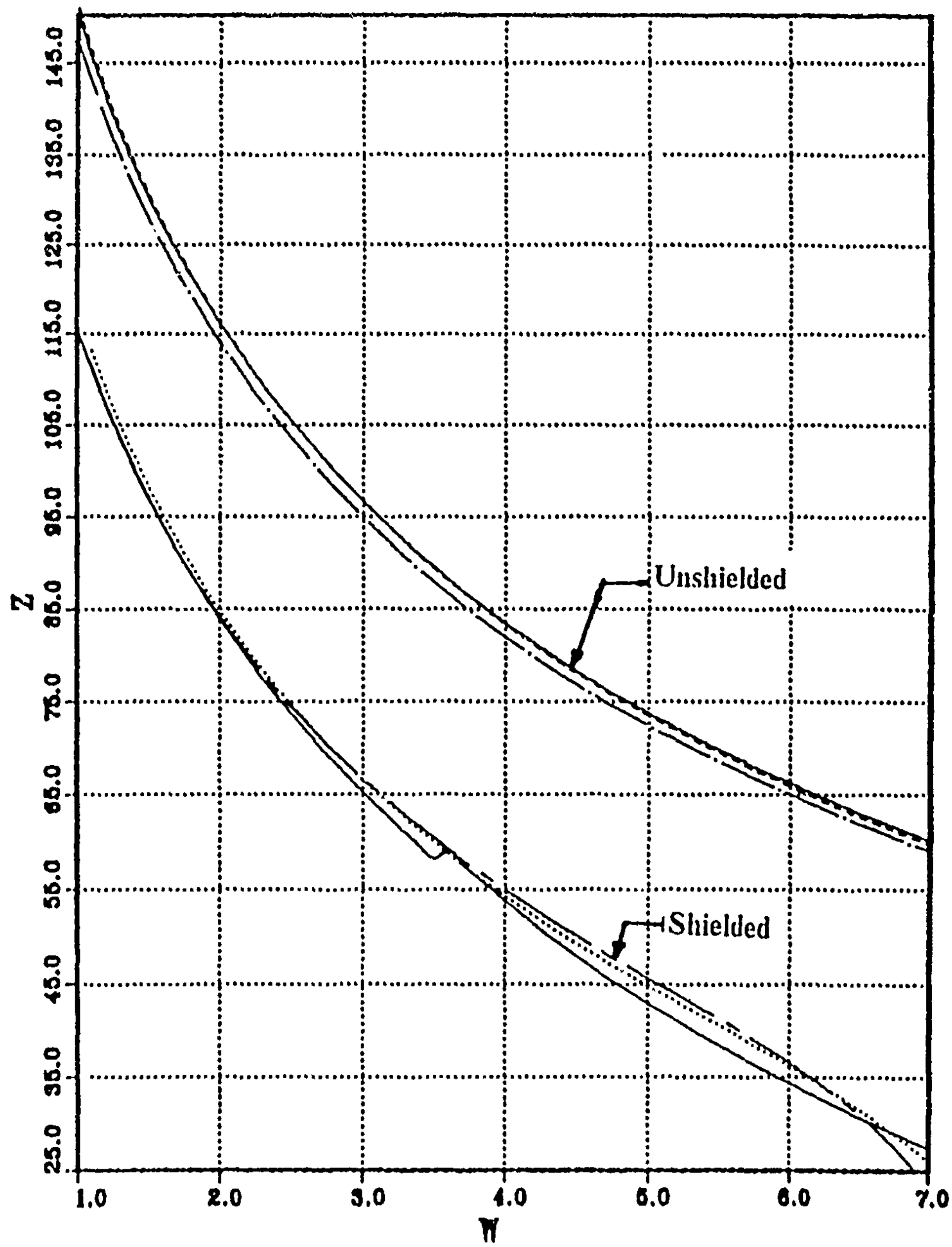

Figure 9. Comparison Between Design Equations for Shielded-Unshielded SSL.

III. DISCONTINUITY GAP CAPACITANCE

A. INTRODUCTION

Since the development of microwave integrated circuits, strip-type transmission lines have been widely used as fundamental structures.

Although a great deal of work has been published on the properties of the suspended strip transmission lines, the theoretical or experimental results have been almost entirely limited to the impedance and the phase velocity.

Since published results for the discontinuity structures in suspended strip transmission lines, such as the gap or an abruptly ended strip conductor, are not now available, especially in the case of shielded SSL, there appears to be a need for quantitative characterization of the discontinuity structures, either in term of equivalent-circuit models, or scattering-parameter characterization.

The purpose of this chapter is to present a theoretical method for calculation of the static capacitance of a series gap in Shielded Suspended Stripline based on Ref. 4 and to show the procedures used for its implementation.

B. CALCULATION OF GAP CAPACITANCE IN SHIELDED SSL [REF. 4]

The physical gap structure in the strip conductor of the microstrip transmission line in shown in Fig. 10.

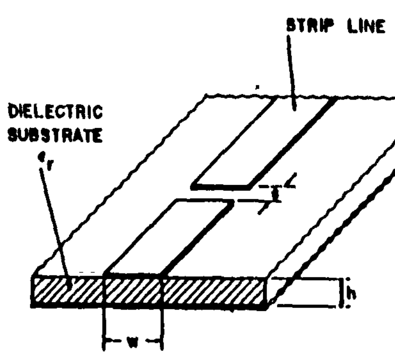

Figure 10. Physical Gap Structure.

The individual gap is expected to have a series capacitance between the open ends of line, plus capacitances to ground (shield) due to the fringing fields at the break in the line. These capacitances can be calculated as follows:

By placing an element of the line in a shield box with 1/2 gap length at each end, as shown in Fig. 11, the gap structure can be represented by an equivalent π-circuit as shown in Fig. 12.

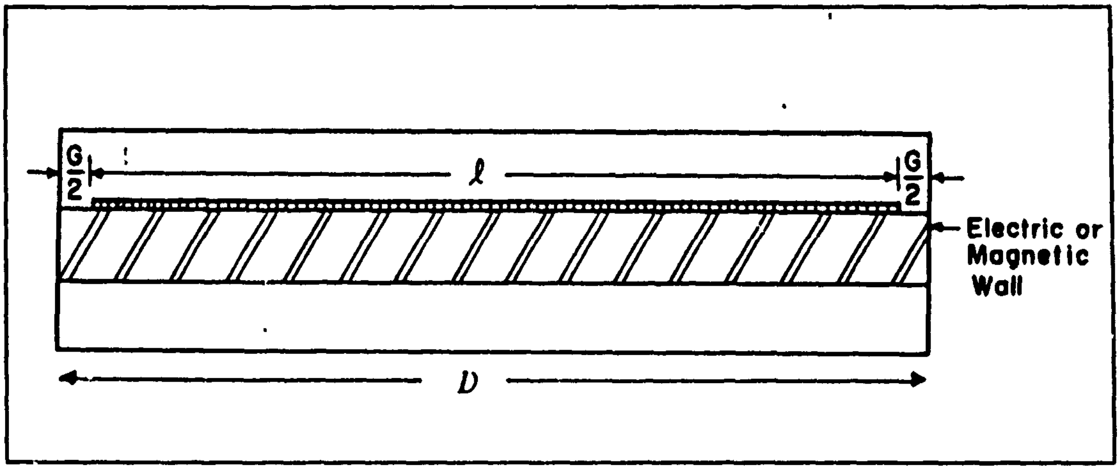

Figure 11. Shield Box.

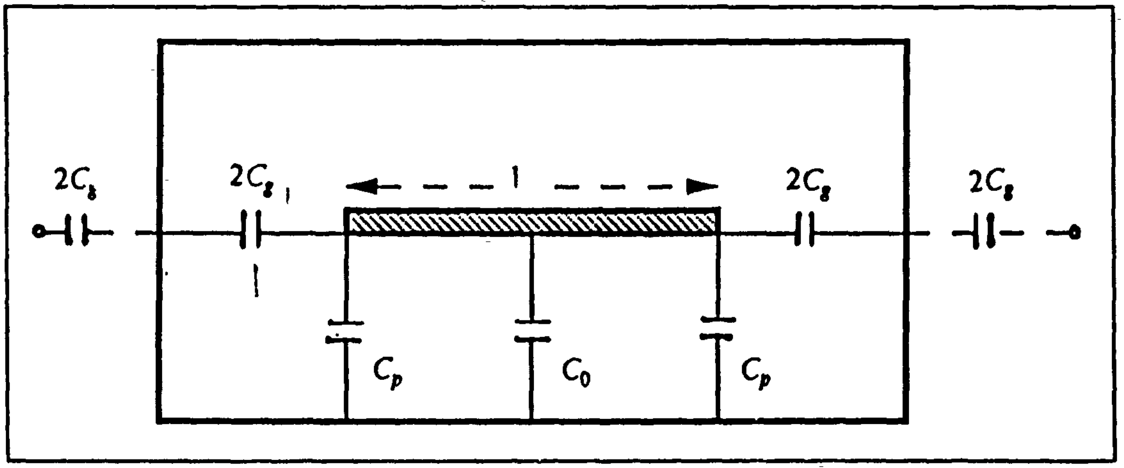

Figure 12. Equivalent π Circuit.

where

C_g = series gap capacitance which represents the electrostatic capacitance between the two open ends of line at the gap. This is represented here by capacitor $2C_g$ and the image in the end wall of the shield, as shown in Fig. 12.

C_0 = line capacitance of the uniform microstrip transmission line with length l.

C_p = parasitic capacitance due to electrostatic field lines extending from the open end to ground.

By finding a way to calculate the total capacitance between the line section and the enclosing shield box for the two cases:

- with electric walls at the end (giving C_E).
- with magnetic walls at the end (giving C_M).

then the gap capacitance C_g and parasitic capacitance C_P can be calculated from:

$$C_g = \frac{C_E - C_M}{4} \tag{34}$$

$$C_P = \frac{C_M - lC_0}{2} \tag{35}$$

The method for obtaining electric and magnetic walls at the end of the box is to assume an electric E-field within the shield which has no component perpendicular to the end walls (magnetic-wall case), with the opposite being true for the electric-wall case. These assumed fields are written in the form of finite-series summations. Each term in the series is separately a solution of Laplace's equation, making the assumed potentials valid electrostatic fields. A separate series is assumed for each of the three regions within the shield: upper air region, the dielectric layer, and the lower region. Each of these series contains constant coefficients with each term, which must be determined. The method of determining the coefficients is to require matching of electrostatic potentials at the upper interface (containing the conductor) and matching of normal D-vectors at the lower dielectric-air interface.

A series is assumed solution for the potential in each of the three regions 1, 2, 3 as shown in Fig. 13.

$$\Phi_3 = \sum_n \sum_m D_{mn} \sinh(k_q(y'' - H_3)) \cos(k_m x) \cos(k_n z) \tag{36}$$

$$\Phi_2 = \sum_n \sum_m [B_{mn} \sinh(k_q y') + C_{mn} \cosh(k_q y')] \cos(k_m x) \cos(k_n z) \tag{37}$$

$$\Phi_1 = \sum_n \sum_m A_{mn} \sinh(k_q y) \cos(k_m x) \cos(k_n z) \tag{38}$$

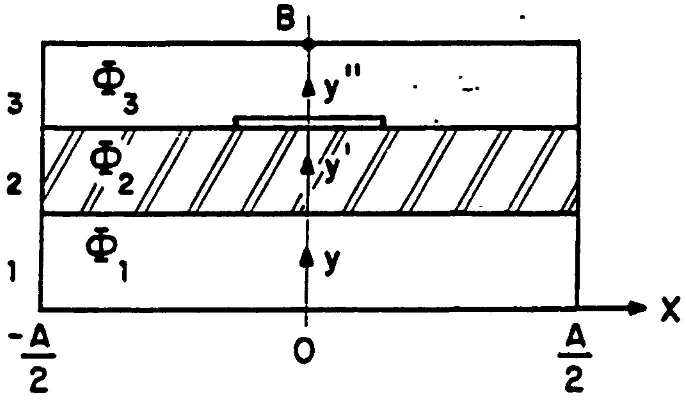

Figure 13. Potential Representation.

where H_n is the air height of region n, and k_q , k_m , and k_n are given below.

With strip line at a potential of 1 Volt and shield at 0 Volts, integrating the D vector over a closed surface surrounding the strip, the capacitance line-to-ground is given by:

$$C = -\varepsilon_0 \int\int \varepsilon_{ri} \nabla \Phi_i ds \tag{39}$$

where i = 1,2,3.

The surface of integration is broken up into the parts covering the regions 1,2 and 3 where the corresponding Φ_i are used.

The coefficients in the Equations (36), (37), and (38) are given as:

$$k_m = \frac{(2m-1)\pi}{A} \tag{40}$$

$$k_n = \frac{(2n-1)\pi}{D} \quad (\textit{Electric wall}) \tag{41}$$

$$k_n = \frac{2n\pi}{D} \quad (\textit{Magnetic wall}) \tag{42}$$

$$k_q = \sqrt{k_m^2 + k_n^2} \tag{43}$$

where D is the length of the shield box as shown in Fig. 11.

Using the following boundary conditions:

- Interface (1)-(2)

$$\Phi_1 = \Phi_2 \tag{44}$$

$$\frac{d\Phi_1}{dy}\ (at\, y = H_1) = \varepsilon_r \frac{d\Phi_2}{dy'}\ (at\, y' = 0) \tag{45}$$

- Interface (2)-(3)

$$\Phi_2 = \Phi_3 \tag{46}$$

$$for \quad |x| \le \frac{W}{2} \qquad \Phi_2 = 1 \tag{47}$$

$$for \quad |x| > \frac{W}{2} \qquad \varepsilon_r \frac{d\Phi_2}{dy'} = \frac{d\Phi_3}{dy''} \tag{48}$$

the coefficients A_{mn}, B_{mn}, C_{mn} can be evaluated in terms of D_{mn} using equations (44)-(48) at the interfaces between layers. Then all potentials are known in terms of the unknown D_{mn}.

To solve for these coefficients we evaluate boundary conditions on a grid of selected points (x_p, z_p) over the interface containing the strip , as shown in Fig. 14.

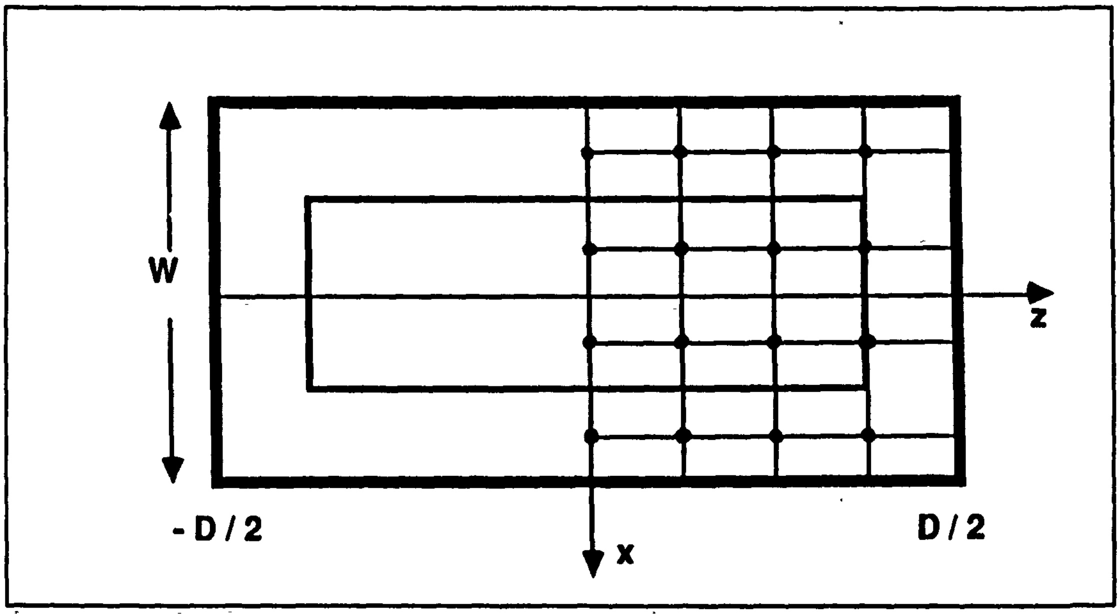

Figure 14. Grid of Points.

As more points are included, the accuracy of the solution increases. The boundary conditions lead to a set of matrix elements to solve for the unknown coefficient D_q, after coefficients D_{mn} are mapped into the single-subscript unknown D_q , as a matrix calculation:

$$[B_{pq}]\cdot[D_q] = [W_p] \tag{49}$$

where

- for x_p, z_p on the metal strip

$$B_{pq} = S_3 \cos(k_m x_p) \cos(k_m y_p) \tag{50}$$

$$W_p = -1 \tag{51}$$

- for x_p, z_p on the air-dielectric boundary

$$B_{pq} = k_q \frac{K_1 S_2 K_3 + \varepsilon_r K_2 S_{13} + \varepsilon_r^2 S_1 S_2 S_3}{K_1 S_2 + \varepsilon_r K_2 S_1} \cos(k_m x_p) \cos(k_n y_p) \tag{52}$$

$$W_p = 0 \tag{53}$$

where

$$K_N = \cosh(k_q H_N) \tag{54}$$

$$S_N = \sinh(k_q H_N) \tag{55}$$

$$S_{13} = \sinh(k_q(H_1 + H_3)) \tag{56}$$

where N = 1,2,3 and the H_N are the heights of the respective regions.

The capacitance integral produces the formula:

$$CAP = 4\varepsilon_0 \sum_m \sum_n \frac{K_1 S_2 K_3 + \varepsilon_r K_2 S_{13} + \varepsilon_r^2 S_1 S_2 S_3}{K_1 S_2 + \varepsilon_r K_2 S_1} P \tag{57}$$

where

$$P = D_q \frac{k_q}{k_m k_n} \sin(k_m \frac{A}{2}) \sin(k_m \frac{D}{2}) \tag{58}$$

C. IMPLEMENTATION

Using the previously described method, a Fortran program was developed for calculation of the gap and the parasitic capacitances of a given filter structure. This program is given in Appendix B.

The basic steps for the program development are:

1. Create a mesh of points (x_i, z_j) (as in Fig. 14) on the (x,z) plane and store them in array $[x,z]$.
2. Compute the coefficients k_n (equation 41) for n = odd and store them in an array $[k_n(n_1)]$, in order to use them for the electric capacitance calculations. Compute k_n for n = even (equation 42) and store them in array $[k_n(n_2)]$ to use for the magnetic capacitance calculations.
3. The array of points x_i, z_j has dimensions I_{max} *by* J_{max}. It is necessary to convert this to one-dimensional string by using:

   $$p = I_{max}(j-1) + i \qquad \textit{for the } (i,j)^{th} \textit{ member} \tag{59}$$

   Similarly the double subscripted D_{mn} has to be converted to a single-subscript quantity D_q, to enable solution of the matrix. This is done by using :

   $$q = m_{max}(n-1) + m \tag{60}$$

4. Compute the coefficients k_m (equation 40) for m = 1 to m_{max} and store them in array $[k_m]$.
5. Compute the coefficients k_q (equation 43) for $k_n(n_1)$ and calculate the arrays $[B(p,q)]$ and $[W(p)]$, on the metal strip and on the air-dielectric boundary.
6. Use Gaussian elimination to solve for the unknown coefficients D_q as : $[D_q] = [B_{pq}]^{-1} [W_p]$.
7. Calculate the electric-wall capacitance using the equation (57).
8. Compute the k_q (equation 43) for $k_n(n_2)$, calculate the arrays $[B(p,q)]$ and $[W(p)]$, and solve to compute the D_q.
9. Calculate the magnetic capacitance using the equation (57).
10. Calculate the gap capacitance using equation (34).
11. Calculate the parasitic capacitance using equation (35).

The Fortran program given in Appendix B was implemented on the Naval Postgraduate School's IBM 3033 computer, for maximum number of points (allowed by the available storage capacity), $I_{max} = 23$, $J_{max} = 23$ and also $n_{max} = 23$ and $m_{max} = 23$.

Because of the limitation of the number of points available, it was necessary to adjust the distribution of the points in the (x,z) plane, in order to have more points in the area of the gap-strip boundary. This was done by using the formulas:

$$Z_J = \frac{D - G}{2J_f} J \quad for \;\; J \leq J_f \tag{61}$$

$$Z_J = \frac{D - G}{2} + \frac{G}{2J_{max}} J \quad for \;\; J > J_f \tag{62}$$

where J_f is the number of points choosen on the strip. For example, for $J_{max} = 23$ and for $J_f = 10$, there are 10 points on the strip and 13 points outside the strip.

Also it was found that, if the distribution of the points is not correct, the Fortran program gives negative values of the electric (C_E) or magnetic (C_M) capacitances.

In this event, the following steps are taken:

- In case of negative electric capacitance, the number of points on the strip (J_F) must be increased.
- In case of negative magnetic capacitance, the number of points outside the strip must be increased (J_f to be decreased).

Implementing the Fortran program with shield dimension D = 12 mm, number of points of 23x23, and using the dimensions of the waveguide of interest (WR-28), the gap

capacitance is calculated for gap dimensions between 0.2 mm to 0.8 mm, as shown in Fig. 15.

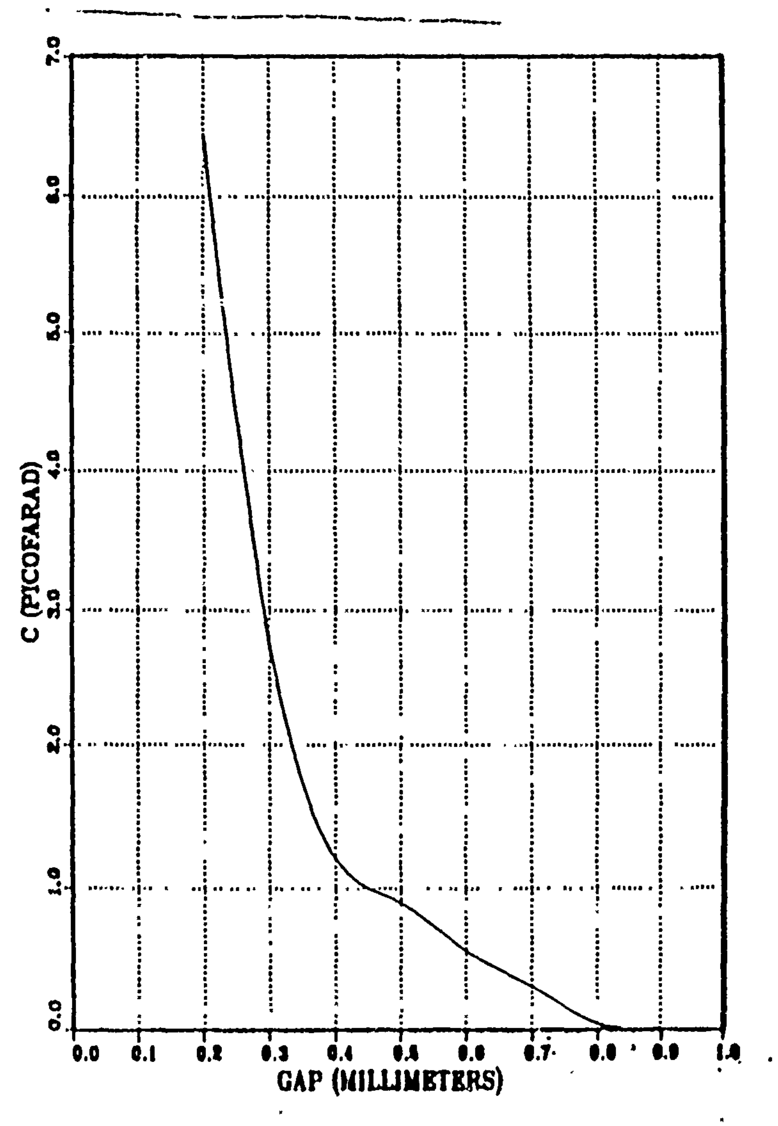

Figure 15. Gap Capacitance Calculations.

D. COMPARISON OF THE RESULTS

In order to check the accuracy of the results given from Fortran program (Appendix B), the capacitances of the SSL structure shown in Fig. 16 were determined.

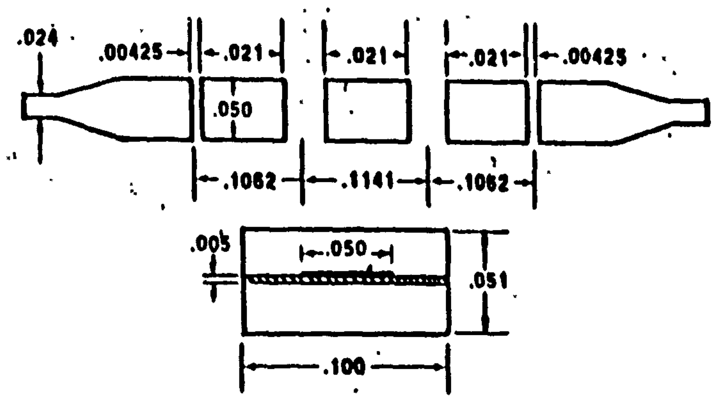

Figure 16. Cross Section of SSL.

The gap capacitances of this structure are given in [Ref. 10: p.75] calculated by using a Fortran program based on the work of Smith [Ref. 11: p. 424] and are given by:

For Gap = 0.107 mm Capacitance = 0.0210 pF

For Gap = 0.307 mm Capacitance = 0.00634 pF

Using the Fortran program given in Appendix B, with dimension of the shield box D = 10 mm (Fig. 11) the calculated values are:

For Gap = 0.107 mm Capacitance = 0.0320 pF

For Gap = 0.307 mm Capacitance = 0.00580 pF

Thus as can be seen the calculated gap capacitances are in reasonable agreement with the values of those in [Ref. 10: p.75], which were also approximate calculations from a static model.

E. CONCLUSION

Considering all of the above, we can conclude that the model given in [Ref. 4] for the calculation of the gap capacitance in shielded SSL structures is relatively easy to implement and gives accurate results.

IV. SHIELDED SUSPENDED STRIPLINE FILTER DESIGN

A. INTRODUCTION

Suspended and inverted microstrip lines are among the principal transmission media used in the upper microwave and lower mm-wave bands. A characteristic aspect of these lines is that the presence of an air gap between the substrate and the ground plane typically reduces the effects of dispersion on the propagation constant to an extent that the quasi-static results remain useful even at very high frequencies.

The suspended strip line (SSL) is a modified version of the microstrip line. Compared with the normal microstrip line, it has some attractive features, such as lower attenuation and larger tolerance of fabrication variances.

B. MICROWAVE FILTER THEORY

The network synthesis methods for filter design, generally start out by specifying a transfer function, as a function of complex frequency p.

The Tchebyscheff and maximally flat transfer functions are often used for filter applications. In this work only the Tchebyscheff design is considered, since is preferred because of its sharp cutoff. The attenuation characteristic of the Tchebyscheff, or "equal ripple" is shown in Fig. 17. The attenuation δ_m is the maximum db attenuation in the pass-band, while ω_1 is the equal-ripple band edge.

Attenuation characteristics of this may be specified mathematically as [Ref. 9: Appendix C].

$$\delta = 10\log(1 + \varepsilon\cos^2(n\cos^{-1}(\frac{\omega}{\omega_1}))) \qquad \omega \le \omega_1 \tag{63}$$

$$n_{ER} = 10\log(1 + \varepsilon\cosh^2(n\cosh^{-1}(\frac{\omega}{\omega_1}))) \qquad \omega \ge \omega_1 \tag{64}$$

$$\varepsilon = (\text{antilog}(\frac{\delta_{AR}}{10})) - 1 = \frac{(\rho_m - 1)^2}{4\rho_m} \tag{65}$$

where: δ = insertion loss, n_{ER} = isolation, and n = number of reactive elements in the circuit. The basic prototype circuit is shown in Fig. 17.

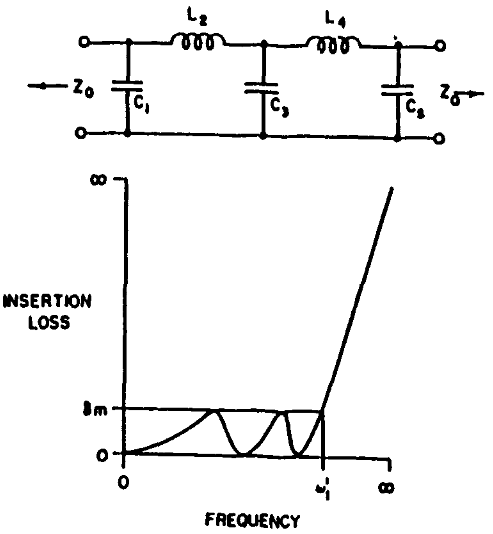

Figure 17. Prototype Circuit [Ref.2, Appendix C].

The element values $g_0, g_1, .., g_n, g_{n+1}$ for a low-pass prototype must alternate between shunt capacitor and series inductor. The structure can begin with either inductor or capacitor. The index associated with each element is given by k. Then the following equations define the element values [Ref. 1: p. 97], [Ref. 2: Appendix C].

$$\frac{\omega_1 C_k}{Y_0} = g_k \tag{66}$$

$$\frac{\omega_1 L_k}{Z_0} = g_k \tag{67}$$

$$g_1 = \frac{2a_1}{\gamma} \tag{68}$$

$$g_k = \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}} \tag{69}$$

$$a_k = \sin(\frac{2k-1}{2n}\pi) \tag{70}$$

$$b_k = y^2 + \sin^2(\frac{k}{n}\pi) \tag{71}$$

$$y = \sinh(\frac{1}{2n}\ln(coth\frac{\delta_m}{17.37})) \tag{72}$$

$$\delta_m = 20log(\frac{\rho_m + 1}{2\rho_m}) = (\rho - 1)^2 \tag{73}$$

where ρ_m is the max VSWR.

C. TRANSFORMATIONS

1. High-Pass Prototype

The low-pass prototype is converted to a high-pass prototype by using series capacitors and shunt inductors [Ref. 9: Appendix C] as shown in Fig. 18.

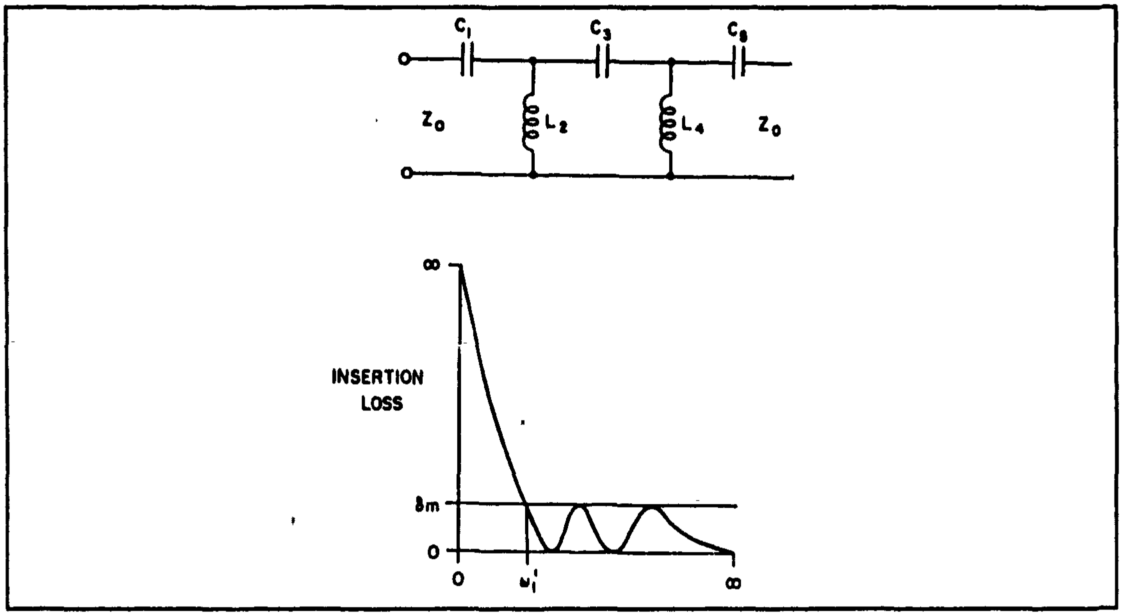

Figure 18. High-Pass Prototype [Ref. 9: Appendix C].

The element values are to be found using the following equations

$$\frac{\omega_1 C_k}{y_0} = \frac{1}{g_k} \tag{74}$$

$$\frac{\omega_1 L_k}{Z_0} = \frac{1}{g_k} \tag{75}$$

Equations (63) and (67) are the same except that attenuation δ is above frequency ω_1 and isolation n is used below frequency ω_1.

2. Band-Pass Prototype

The low-pass prototype filter is converted to a band-pass filter [Ref. 9: Appendix C] simply by resonating all of the elements at the center (geometric mean) frequency of the pass-band.

First the ripple, bandwidth and number of elements are selected, to give the maximum insertion loss and bandwidth desired in the pass-band. This produces a low-pass prototype as in Fig. 17.

Then given upper frequency ω_U and lower frequency ω_L the center frequency ω_0 is calculated

$$\omega_0 = \sqrt{\omega_U \omega_L} \tag{76}$$

Then inductors are selected to parallel resonate with the shunt capacitors as

$$L_1 = \frac{1}{\omega_0^2 C_1} \quad C_2 = \frac{1}{\omega_0^2 L_2} \quad etc. \tag{77}$$

The resonating elements simply shift the response up to a higher frequency.

D. CAPACITIVE-GAP-COUPLED TRANSMISSION LINE FILTERS

The SSL filter considered here is a capacitive-gap-coupled transmission line filter [Ref. 8: p. 441].

Figure 19 shows the structure of a coupled-resonator filter consisting of transmission-line resonators which are approximately a half wavelength long at the midband frequency ω_0 and which have series capacitance coupling between resonators.

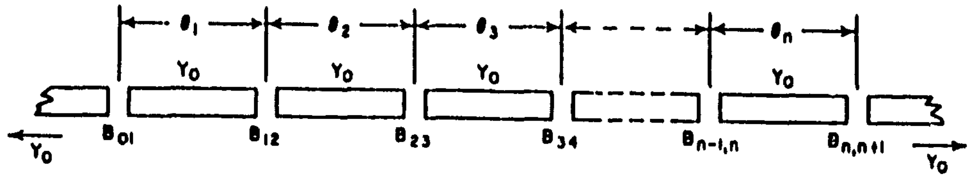

Figure 19. Capacitive-Gap Coupled Filters [Ref. 1: p. 441].

In this case the inverters are in the form of series capacitive types. These inverters tend to reflect high impedance levels to the ends of the half-wavelength resonators and it can be shown that this causes the resonators to exhibit a shunt-type resonance [Ref. 8: p. 440]. Thus the filters under consideration operate like the shunt-resonator type of filter whose general design equations [Ref. 8: p. 440] are:

$$\frac{J_{0,1}}{Y_0} = \sqrt{\frac{\pi}{2}\frac{\omega}{g_0 g_1 \omega'_1}} \tag{78}$$

$$\frac{J_{j,j+1}}{Y_0} = \frac{\pi W}{2\omega'_1}\frac{1}{\sqrt{g_j, g_{j+1}}} \quad for\, j = 1\ to\ n-1 \tag{79}$$

$$\frac{J_{n,n+1}}{Y_0} = \sqrt{\frac{\pi W}{2 g_n g_{n+1} \omega'_1}} \tag{80}$$

where

$g_0, g_1, \ldots, g_n$: Tchebyscheff coefficients [Ref. 8: p. 100].

W : the fractional bandwidth

$J_{j,j+1}$: admittance inverter parameters

Y_0 : characteristic admittance of the line

Assuming the capacitive gaps function as ideal series-capacitance discontinuities of susceptance $B_{j,j+1}$ we have:

$$\frac{B_{j,j+1}}{Y_0} = \frac{\frac{J_{j,j+1}}{Y_0}}{1 - (\frac{J_{j,j+1}}{Y_0})^2} \tag{81}$$

$$\theta_j = \pi - \frac{1}{2}\left[\tan^{-1}(\frac{2B_{j-1,j}}{Y_0}) + \tan^{-1}(\frac{2B_{j,j+1}}{Y_0})\right] \quad \textit{in radians} \tag{82}$$

where $B_{j,j-1}$ and θ_j are evaluated at ω_0

Using the above susceptance values the gap-capacitances are derived using the following equation:

$$\Delta C_{j,j+1} = (\frac{B_{j,j+1}}{Y_0})\, Y_0\, \frac{1}{2\pi f_0} \tag{83}$$

E. SUSPENDED STRIPLINE FILTER DESIGN

The filter design parameters selected for this design are

- Center frequency = 33.25 Ghz
- Bandwidth = 2.6 Ghz
- Percentage Bandwidth = 8%
- Number of poles = 3 and 5
- Ripple = 0.2 db

The design of the filter consists of two major parts:

- Calculation of the transmission line parameters of the filter line elements.
- Calculation of the gaps and the resonance lengths of the filter.

1. Calculation of the Transmission Line Parameters of the Shielded SSL

For the waveguide of interest WR-28 (A = 7.11 mm, B = 3.56 mm), with $\varepsilon_r = 2.22$, h = 0.5 mm and Z = 60 Ω, using the equations (2) and (14) we calculate:

W = 3.33 mm

$\varepsilon_e = 1.093769$ where ε_e = effective dielectric constant. So finally the filter structure is as shown in cross section in Fig. 20.

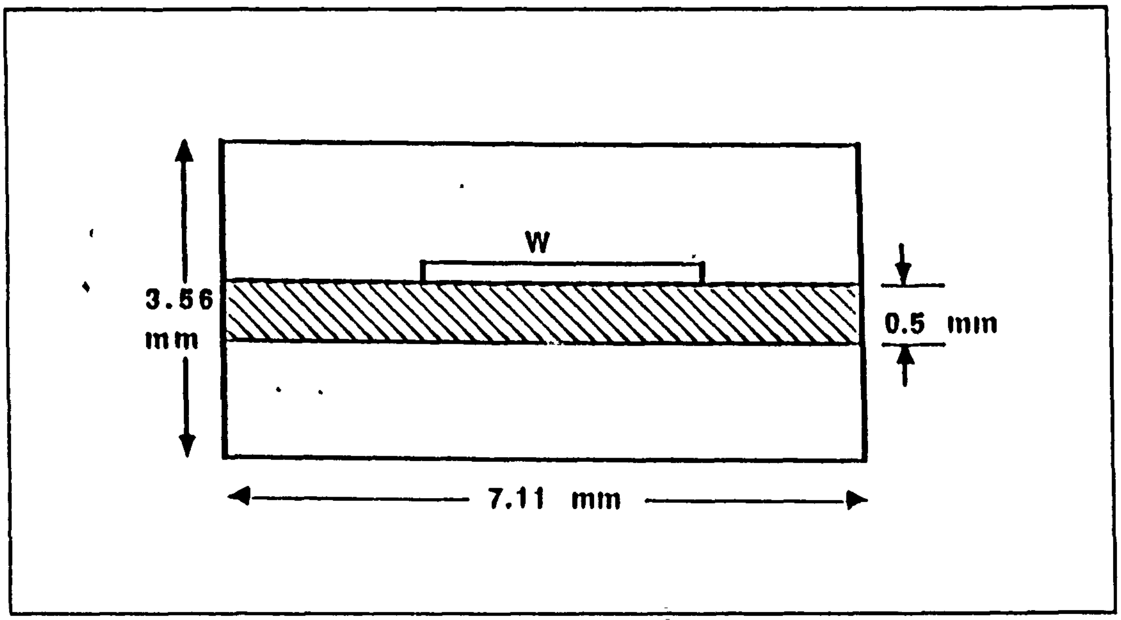

Figure 20. SSL Filter Cross-Sectional View.

2. Calculation of the Gaps and Resonance Lengths

a. *Three Element Filter*

For N = 3 and r = 0.2db and using the coefficients from [Ref. 8: p. 100]

$g_1 = g_3 = 1.2275, g_2 = 1.1525, g_4 = 1$

Using the design equations (78) through (83), with the following specifications:

- Bandwidth BW = 0.08
- Characteristic impedance Z = 60 Ω
- Center Frequency = 33.25 Ghz
- Effective dielectric constant k = 1.093769

we calculate:

- Gap capacitances

$$C_{01} = 0.28436\ pF$$
$$C_{12} = .08523\ pF$$
$$C_{23} = .08523\ pF$$
$$C_{34} = 0.28436\ pF$$

- The length of each element of the filter in radians

$$\theta_1 = 2.7266600\ rad$$
$$\theta_2 = 2.9310686\ rad$$
$$\theta_3 = 2.7266600\ rad$$

- The actual length of each element in mm

$$L_1 = 3.772\ mm$$
$$L_2 = 4.055\ mm$$
$$L_3 = 3.772\ mm$$

- Using Fig.15 in Chapter 3 we calculate the required gaps for the previously specified gap capacitances.

$$\Delta_{01} = 0.70\ mm$$
$$\Delta_{12} = 0.75\ mm$$
$$\Delta_{23} = 0.70\ mm$$

- Using the Fortran program in Appendix B we calculate the parasitic capacitances as follows:

$$C_{par01} = 0.44\ 10^{-15}\ F$$
$$C_{par12} = 0.20\ 10^{-16}\ F$$
$$C_{par23} = 0.44\ 10^{-15}\ F$$

b. Five Element Filter

For $N = 5$ and $r = 0.2$ db we pick the coefficients from [Ref. 8: p. 100]

$$g_1 = g_5 = 1.3394,\ g_2 = g_4 = 1.3370,\ g_3 = 2.1660,\ g_6 = 1.3394,\ g_7 = 1.0$$

Using the design equations (78) through (83) we calculate:

- Gap capacitances

$$C_{01} = 0.2696\ pF$$
$$C_{12} = 0.0755\ pF$$
$$C_{23} = 0.0592\ pF$$
$$C_{34} = 0.0592\ pF$$
$$C_{45} = 0.0755\ pF$$
$$C_{56} = 0.2696\ pF$$

- The lengths of each element in rad

$$\theta_1 = 2.7507\ rad$$
$$\theta_2 = 2.9742\ rad$$
$$\theta_3 = 2.9941\ rad$$
$$\theta_4 = 2.9742\ rad$$
$$\theta_5 = 2.7507\ rad$$

- The actual length of each element in mm

$$L_1 = 3.776\ mm$$
$$L_2 = 4.083\ mm$$

$L_3 = 4.111\ mm$
$L_4 = 4.083\ mm$
$L_5 = 3.776\ mm$

- Using Fig. 15 in Chapter 3 we calculate the required gaps for the previously specified gap capacitances:
$\Delta_{01} = 0.70\ mm$
$\Delta_{12} = 0.75\ mm$
$\Delta_{23} = 0.80\ mm$
$\Delta_{34} = 0.80\ mm$
$\Delta_{45} = 0.75\ mm$
$\Delta_{56} = 0.70\ mm$

- Using the Fortran program given in Appendix B we calculate the parasitic capacitances for each case as follows:
$C_{par01} = 0.440\ 10^{-15}\ F$
$C_{par12} = 0.830\ 10^{-15}\ F$
$C_{par23} = 0.166\ 10^{-16}\ F$
$C_{par34} = 0.166\ 10^{-16}\ F$
$C_{par45} = 0.830\ 10^{-15}\ F$
$C_{par56} = 0.440\ 10^{-15}\ F$

Considering all the above calculations the complete filter sructures are as shown in Fig. 21 and Fig. 22.

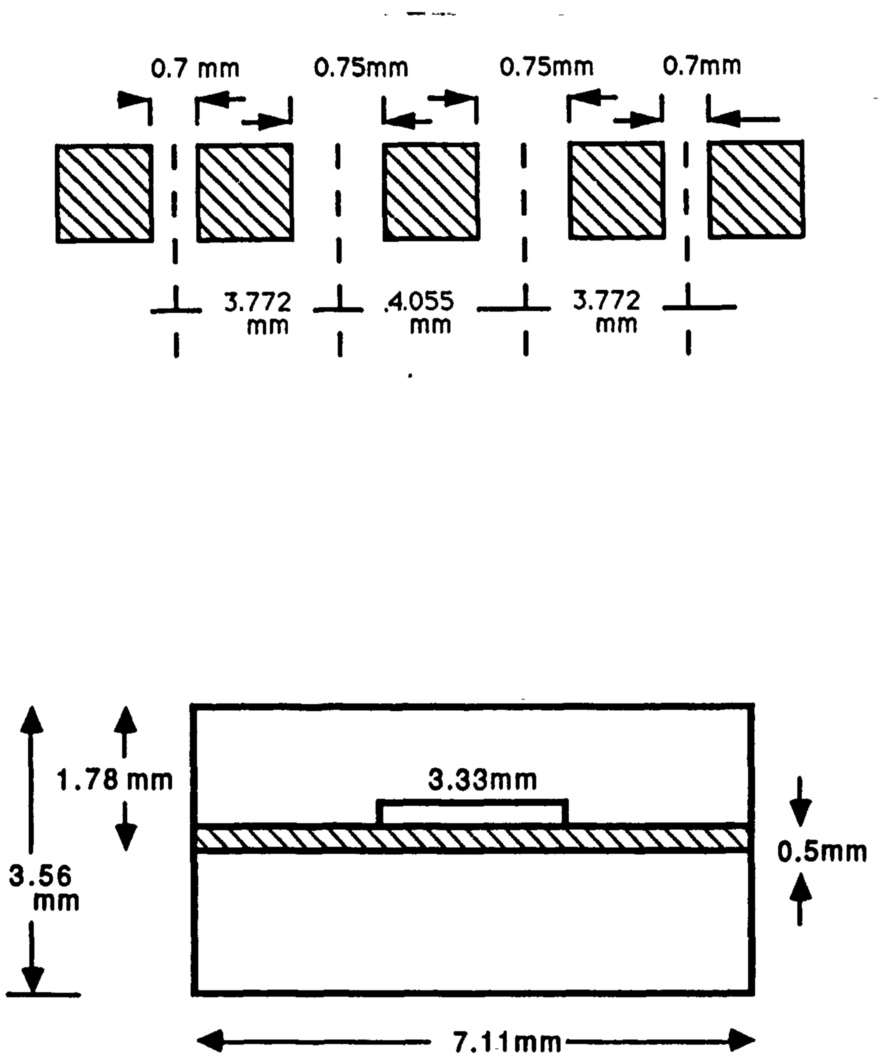

Figure 21. Complete 3 Element Filter Structure.

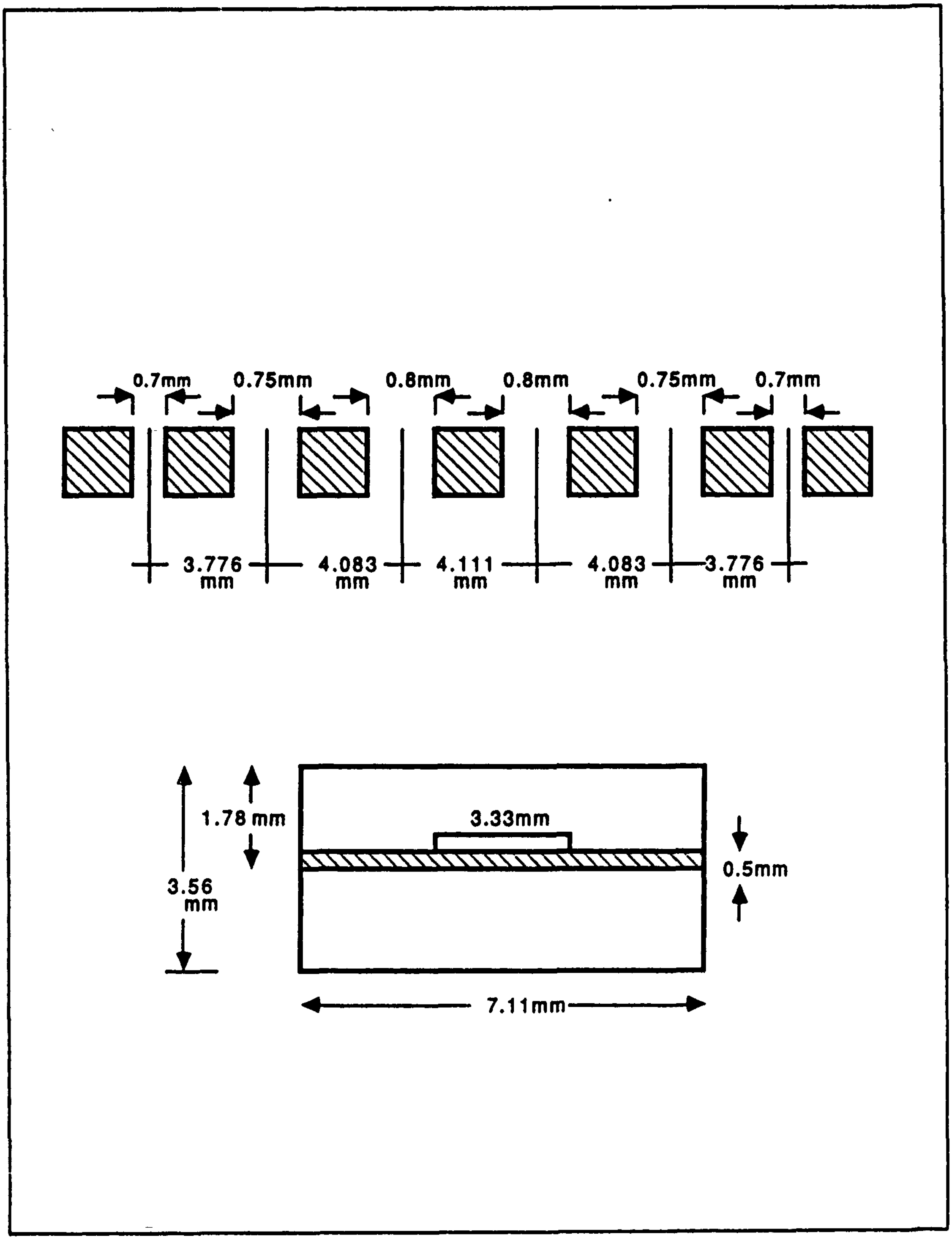

Figure 22. Complete 5 Element Filter Structure.

F. FILTER EVALUATION

Using the previously calculated values, the two filters were characterized using the Touchtone EEsof's computer-aided microwave simulation and optimization program (available from the EEsof Corporation, Westlake Village California). The models used in the Touchstone programs are shown in the next Figs. 23 and 24.

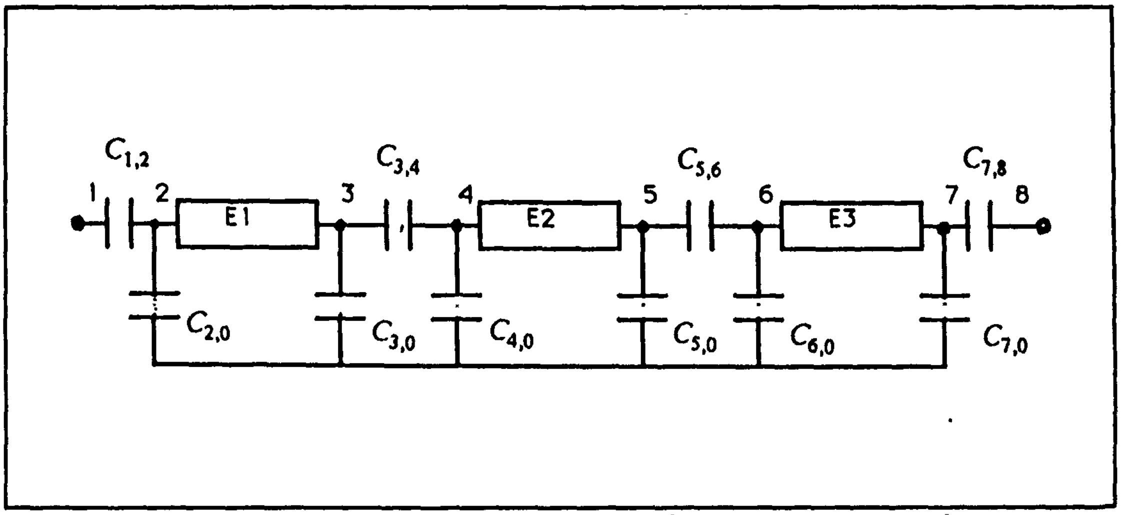

Figure 23. 3 Element Filter Structure.

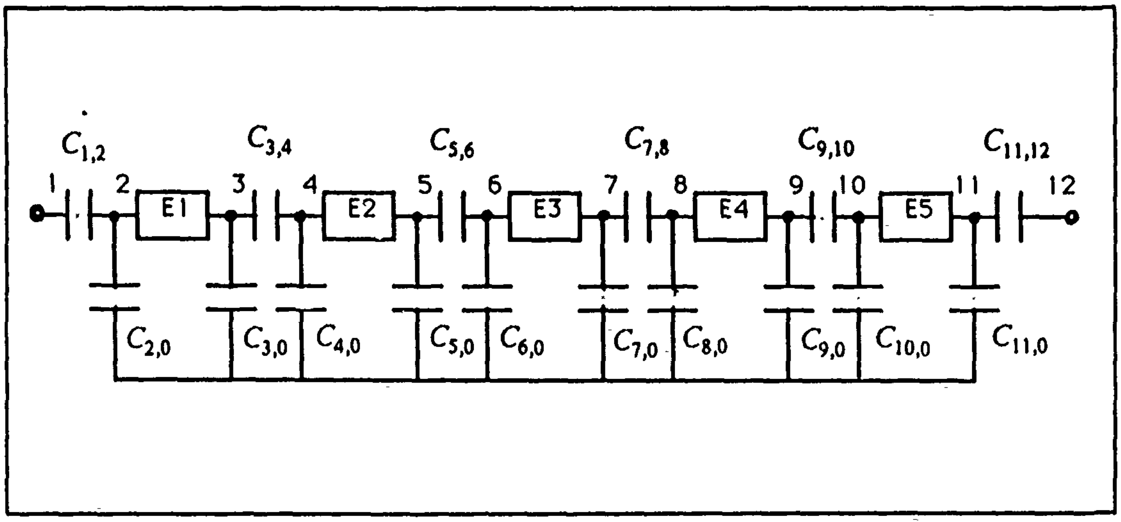

Figure 24. 5 Element Filter Structure.

From the TOUCHSTONE calculation the frequency responses of the two filters are plotted as shown in Figs. 25 and 26. As shown in those two figures the filter bandwidth and the center frequencies are in agreement with the design values.

The effect of the parasitic capacitances, because of their very small values, was negligible in the frequency response of the filter.

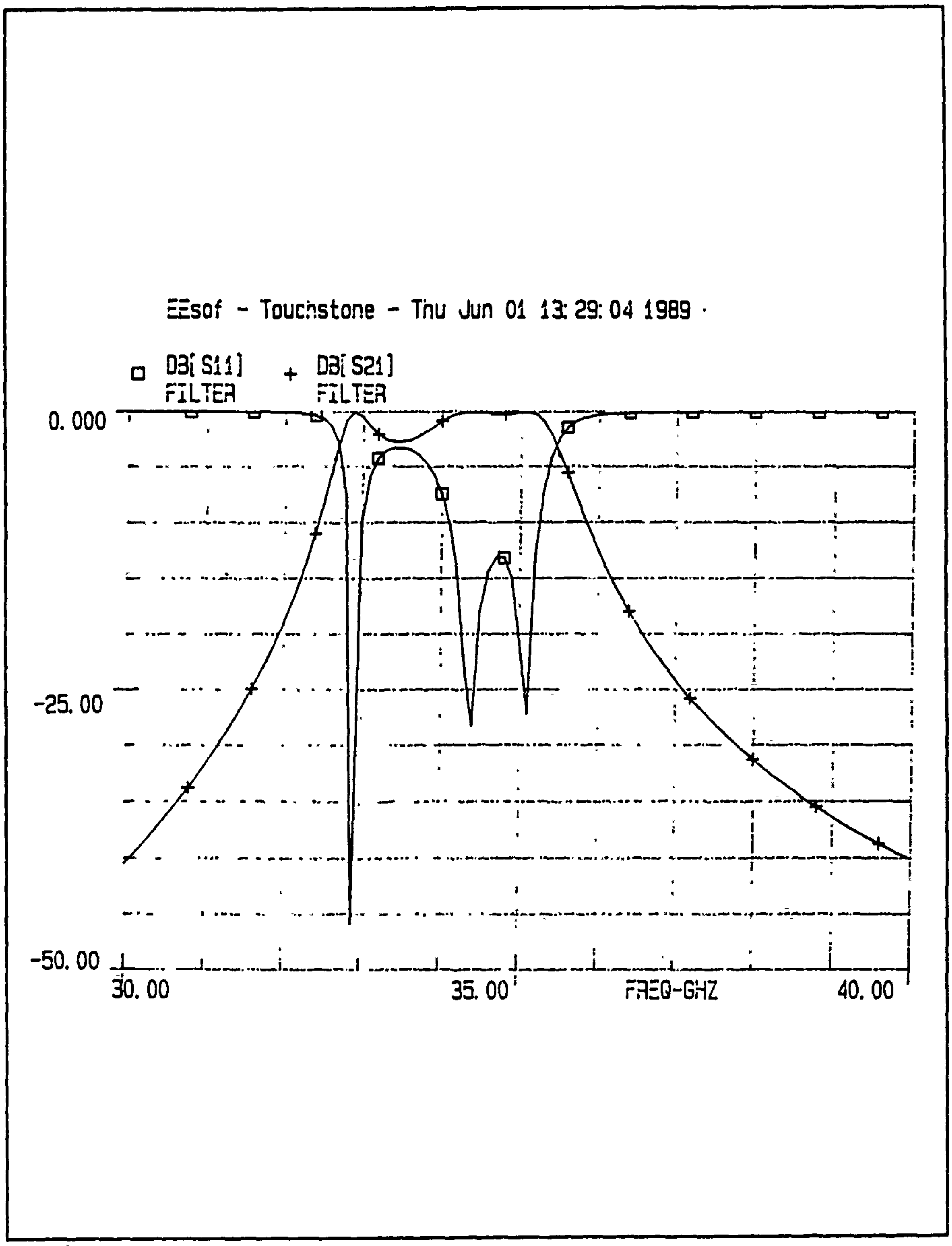

Figure 25. Frequency Response of 3 Element Filter.

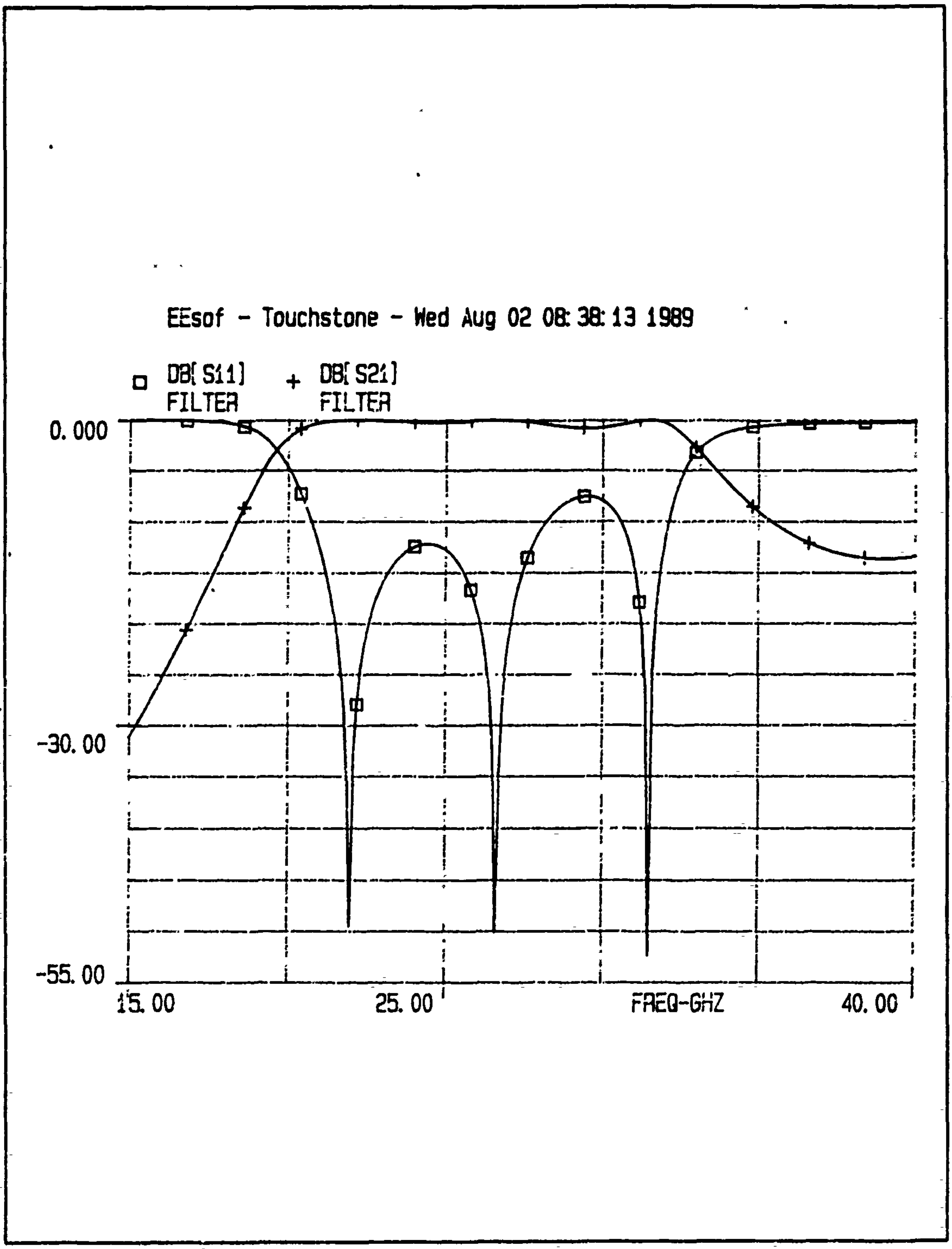

Figure 26. Frequency Response of 5 Element Filter.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The purpose of this research was to establish design rules for the shielded form of Suspended Substrate Stripline as a propagation medium, and also to establish a practical model for calculating the gap capacitance of the filter structure.

The objective of this research was accomplished. The final results are the following:

1. For the calculation of the Shielded SSL transmission line parameters, the Analysis [Ref. 1: p.693] and Synthesis [Ref. 2: p.331] design equations can be used because these relations are useful and accurate.
2. The method given in [Ref. 4] for the calculation of the static capacitance of a series gap in shielded SSL is relatively easy to implement and gives accurate results.

B. RECOMMENDATIONS

Based on the results of this study the following recommendations are made:

1. The dispersive (frequency-dependent) effects must be addressed, especially when the Suspended Stripline filter is to be used for microwave frequencies higher than 10 Ghz. For this purpose a spectral-domain analysis of the propagation in this line system should be made, based on wave potentials which are solutions of the Helmholtz equation in the guiding region.
2. The gap-capacitance calculations should similarly be carried out in a frequency-dependent spectral domain form. Ultimately, other discontinuities, including the line open end and corner (bend) should be investigated.

APPENDIX A. DESIGN EQUATIONS

A. VARIATIONAL CALCULATION METHOD

This is a general method [Ref. 3: p. 238] for analyzing the transmission line characteristics of SSL, with rectangular outer conducting boundaries. This method uses a Green's function for formulating the problem in a variational approach for obtaining practical solutions.

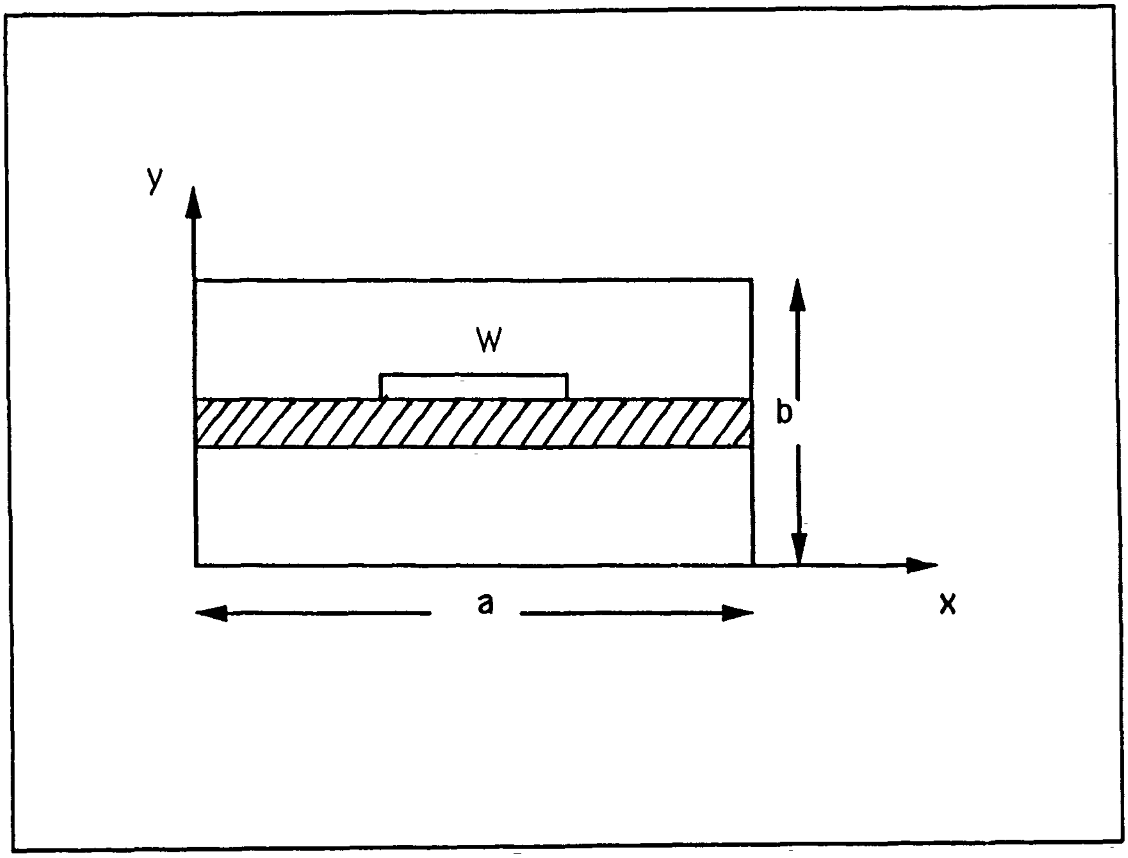

Figure 27. Cross Section of SSL.

The characteristic impedance Z_0 of the line is given by

$$Z_0 = \frac{1}{V_0 \sqrt{CC_0}} \tag{85}$$

The waveguide wavelength λ is

$$\lambda = \lambda_0 \sqrt{\frac{C_0}{C}} \tag{86}$$

and the wave propagation velocity V is given:

$$V = V_0 \sqrt{\frac{C_0}{C}} \tag{87}$$

where:

V_0 = velocity of light in free vacuum space

λ_0 = wavelength in free vacuum space

C_0 = line capacitance for the case $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_0$

The line capacitance is given :

$$C = \frac{[\int f(x)dx]^2}{\int\int f(x)G(x, h_1 + h_2 \mid x_0, h_1 + h_2)f(x_0)dxdx_0} \tag{88}$$

where f(x) is the charge density distribution across the strip conductor and G(x,y) is the Green's function which is regarded as the potential at (x,y), due to a unit charge in an infinitely small volume at (x_0, y_0).

The charge density of thin strip conductor is known to be large at both edges, relative to that at the center. For this reason the following form of the trial function is assumed in this method:

$$f(x) = \frac{1}{W}[1 + K \mid \frac{2}{W}(x - \frac{a}{2}) \mid^3] \quad for \ \frac{a}{2} - \frac{W}{2} \leq x \leq \frac{a}{2} + \frac{W}{2} \tag{89}$$

$$= 0 \qquad \text{otherwise}$$

The constant K is chosen such that to maximize the value of the capacitance C.

The line capacitance is evaluated by applying the equation (89) in equation (88) and is given by:

$$C = \frac{2\varepsilon_0 Q^2}{a \sum_{n=1}^{\infty} V_n^2 g_n} \tag{90}$$

where:

$$V_n = \frac{1}{W}(V_{1n} + kV_{2n}) \tag{91}$$

$$Q = 1 + \frac{K}{4} \tag{92}$$

$$g_n = \frac{a\eta_n}{n\pi\Delta_n} \sinh(\frac{n\pi h_3}{a}) \tag{93}$$

$$V_{1n} = \frac{4}{n\pi} \sin(\frac{n\pi}{2}) \sin(\frac{n\pi W}{2a}). \tag{94}$$

$$\begin{aligned} V_{2n} = \frac{2W}{a} \sin(\frac{n\pi}{2})[&\frac{\sin(\frac{n\pi W}{2a})}{(\frac{n\pi W}{2a})} + \frac{3}{(\frac{n\pi W}{2a})^2} \\ &[\cos(\frac{n\pi W}{2a}) - \frac{2\sin(\frac{n\pi W}{2a})}{(\frac{n\pi W}{2a})} + \frac{\sin^2(\frac{n\pi W}{4a})}{(\frac{n\pi W}{4a})^2}]] \end{aligned} \tag{95}$$

$$\eta_n = \varepsilon_1^* \cosh(\frac{n\pi h_1}{a}) \sinh(\frac{n\pi h_2}{a}) + \varepsilon_2^* \sinh(\frac{n\pi h_1}{a}) \cosh(\frac{n\pi h_2}{a}) \tag{96}$$

$$\Delta_n = \varepsilon_2^* \xi_n \sinh(\frac{n\pi h_3}{a}) + \varepsilon_3^* \eta_n \cosh(\frac{n\pi h_3}{a}) \tag{97}$$

$$\xi_n = \varepsilon_1^* \cosh(\frac{n\pi h_1}{a}) \cosh(\frac{n\pi h_2}{a}) + \varepsilon_2^* \sinh(\frac{n\pi h_1}{a}) \sinh(\frac{n\pi h_2}{a}) \tag{98}$$

The value of the constant K is given by:

$$K = -\frac{\sum_{n=1}^{\infty}(4V_{2n} - V_{1n})V_{1n}g_n}{\sum_{n=1}^{\infty}(4V_{2n} - V_{1n})V_{2n}g_n} \tag{99}$$

Implementation: The described method was applied to a specific waveguide of interest (WR-28) with dimensions a = 7.11 mm and b = 3.56 mm, substrate h = 0.5 mm and ε_r = 2.22. Equation (99) from Ref. 3, p. 238 was not used because it was found that it gave unreasonable values (some negative) of K. Instead of equation (99) the K was chosen such as to maximize the line capacitance C (in this case K = 3).

B. ALTERNATE ASSUMPTION FOR CHARGE DISTRIBUTION

This version [Ref. 4] of the variational calculation method assumes the charge density function across the strip to be simply a step function as shown on Fig. 28.

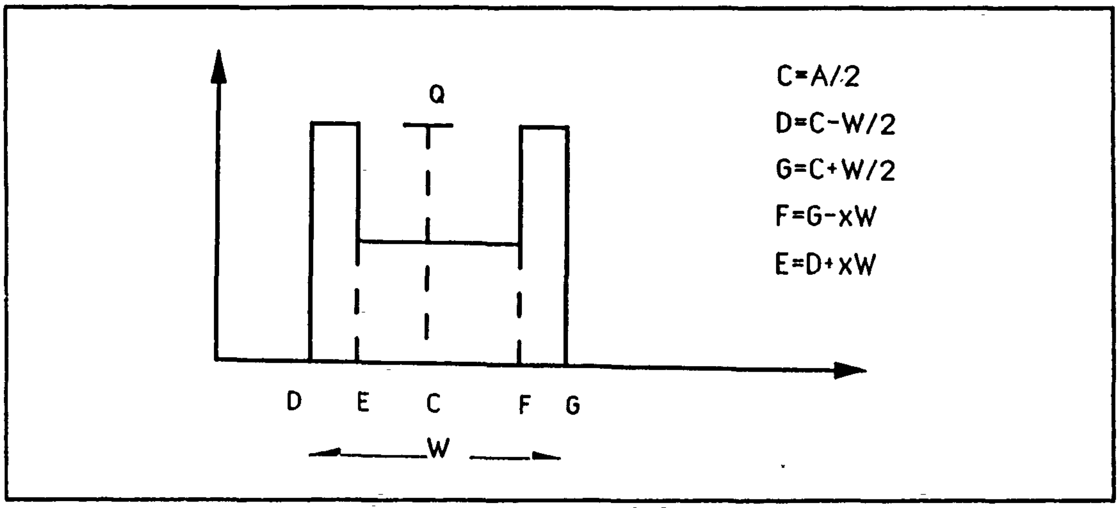

Figure 28. Charge Density Distribution.

By proper choice of the pulse-function height and width at the edges, a good representation of the typical charge singularity which occurs at the edges of strip conductors was obtained. The ratio of the width of charge peaks to linewidth is defined as:

$$\sigma = \frac{FG}{DG} \tag{100}$$

The line capacitance is given by:

$$C = \frac{[\int f(x)dx]^2}{\int\int f(x)G(x, h_1 + h_2 \mid x_0, h_1 + h_2)f(x_0)dxdx_0} \tag{101}$$

On the upper surfaces ($y \geq h_1 + h_2$) the variation calculation method gives:

$$G(x, x_0, y) = \sum_n D_n \sinh(\frac{n\pi}{a}(b-y)) \sin(\frac{n\pi x}{a}) \sin(\frac{n\pi x_0}{a}) \quad . \tag{102}$$

Define the quantity:

$$E_n(n) = D_n \sinh(\frac{n\pi}{A}(b-y)) \tag{103}$$

then

$$G(x, x_0, y) = \sum_n E_n(y) \sin(\frac{n\pi x}{a}) \sin(\frac{n\pi x_0}{a}) \tag{104}$$

Because of the symmetry of the Green's function in x and x_0 , the denominator in Equation(101) becomes:

$$\begin{aligned} Den &= \sum_n E_n(y) \int_{-a/2}^{a/2} f(x) \sin(\frac{n\pi x}{a})dx \int_{-a/2}^{a/2} f(x_0) \sin(\frac{n\pi x_0}{a})dx_0 \\ &= \sum_n E_n(y)[\int f(x) \sin(\frac{n\pi x}{a})dx]^2 \end{aligned} \tag{105}$$

where from the variational calculation method

$$E_n = D_n \sinh(\frac{n\pi}{a}(b-y)) = \frac{2\eta_n}{n\pi\varepsilon_0\Delta_n} \sinh(\frac{n\pi}{a}(b-y)) \tag{106}$$

and η_n, Δ_n, ξ_n are given by equations (96), (97), (98).

Consindering f(x) as in Fig. 28, the integral in equation (105) becomes:

$$
\begin{aligned}
Int &= \int_0^a f(x)\sin\left(\frac{n\pi x}{a}\right)dx = \\
&= -\left(\frac{A}{n\pi}\right)\left[(Q-1)\left(\cos\left(\frac{n\pi E}{a}\right)-\cos\left(\frac{n\pi D}{a}\right)\right)+\right. \\
&\quad \left.+\left(\cos\left(\frac{n\pi G}{a}\right)-\cos\left(\frac{n\pi D}{a}\right)\right)\right]
\end{aligned}
\tag{107}
$$

and

$$
\int f(x)dx = W[1+2\sigma(Q-1)] \tag{108}
$$

Design approach: Considering all the above the design approach is

$$
C = \frac{num}{denom} \tag{109}
$$

$$
Denom = \sum_{n=1}^{\infty} E_n(y)(Int)^2 \tag{110}
$$

$$
\begin{aligned}
Int = -\left(\frac{a}{n\pi}\right)&\left[(Q-1)\cos\left(\frac{n\pi E}{a}\right)-\cos\left(\frac{n\pi F}{a}\right)+\right. \\
&\left.+Q\left(\cos\left(\frac{n\pi G}{a}\right)-\cos\left(\frac{n\pi D}{a}\right)\right)\right]
\end{aligned}
\tag{111}
$$

$$
Num = \{W[1+2\sigma(Q-1)]\}^2 \tag{112}
$$

$$
E_n(y) = D_n \sinh\left(\frac{n\pi}{a}(b-y)\right) \tag{113}
$$

$$
D_n = \frac{2\eta_n}{n\pi\varepsilon_0\Delta_n} \tag{114}
$$

η_n as given in Equation (96)
ξ_n as given in Equation (98)
Δ_n as given in Equation (97)

$$
C = \frac{a}{2} \tag{115}
$$

$$
D = C - \frac{W}{2} \tag{116}
$$

$$E = D + \sigma W \tag{117}$$

$$G = C + \frac{W}{2} \tag{118}$$

$$F = G - \sigma W \tag{119}$$

The values of Q and σ must be chosen in such a way to maximize the calculated capacitance value.

Implementation: This method was applied to a specific waveguide of interest (WR-28) with dimensions a = 7.11 mm, b = 3.56 mm, $h_2 = 0.5$ mm, and $\varepsilon_r = 2.22$.

The parameters Q and σ, were chosen in such a way to maximize the calculated capacitance and for this specific case had the values Q = 22 and $\sigma = 0.01$.

APPENDIX B. FORTRAN PROGRAM

```
C     THIS PROGRAM CALCULATES THE GAP AND THE PARASITIC CAPACITANCES
C     OF A GIVEN FILTER STRUCTURE, USING THE METHOD DESCRIBED IN
C     CHAPTER 3.
c
      DIMENTION X(40),Y(40),KM(40),KN(60),B(485,485),D(485),W1(485)
      real A,B1,H1,H2,H3,D1,G,ER,W,PI,F1,F2,K1,K2,K3,S1,S2,S3,S13
      REAL G1,G2,G3,HH1,T,KQ,S6,C1M,C2M,C3M,C4M,NCAPM,CAPM
      REAL C1E,C2E,C3E,C4E,NCAPE,CAPE,CO
      INTEGER IMAX,MMAX,NMAX,JMAX,I,J,N1,N2,M2,M,N,I2,J2,I3,J3,L
      INTEGER P,Q,K,V,N3,M3,Q1,Q2,N4,N5,N6,M5,N8,I4,J4,I5,J5,IP,JP
 c    A=width of waveguide
 c    B1=length of waveguide
 c    H2=substrate height
 c    W=stripwidth
 c    ER=dielectric constant
 c    CO=capacitance with the dielectric layer replaced by air
 c    D1=length of the shield box
 c    G=gap dimentions
c     Imax=number of points in z-axis
c     Jmax=number of points in x-axis
c     Mmax,Nmax=summation terms
C     *******INPUTS *********
      A=7.11/1000
      B1=3.56/1000
      H2=0.5/1000
      W=3.33/1000
      ER=2.22
C
      CO=
      D1=20/1000
      G=1.0/1000
      IMAX=16
C *** BALANCE BETWEEN THE POINTS ***
C     IP=10
      JP=8
C ***********************************
      JMAX=IMAX
      MMAX=JMAX
      NMAX=IMAX
      PI=3.1415926
      H1=(B1-H2)/2
      H3=H1
      F1=W/2
      F2=(D1-G)/2
C
      V=IMAX*JMAX
C     *** Balance in x-axis ***
      DO 50 I=1,IMAX
C      IF (I .LE. IP) THEN
C       X(I)=((A-W)/(2*IP))*I
C      ELSEIF (I .GT. IP) THEN
```

```
C        X(I)=W/2+((A-W)/(2*IMAX))*I
C       ENDIF
        X(I)=A*I/(2*IMAX)
c       *** Balance in z-axis ***
        DO 45 J=1,JMAX
         IF (J .LE. JP) THEN
          Y(J)=((D1-G)/(2*JP))*J
         ELSEIF (J .GT. JP) THEN
          Y(J)=((D1-G)/2)+(G/(2*JMAX))*J
         ENDIF
   45   CONTINUE
   50  CONTINUE
C      **Calculation of coefficiets Km and Kn for n=even and odd**
       DO 100 M=1,MMAX
        KM(M)=(2*M-1)*PI/A
        DO 80 N=1,NMAX
         N1=2*N-1
         N2=2*N
         KN(N1)=(2*N1-1)*PI/D1
         KN(N2)=2*N2*PI/D1
   80   CONTINUE
  100  CONTINUE
C      **Calculation of coefficients B(p,q) and W(p)**
       DO 300 I2=1,IMAX
        DO 250 J2=1,JMAX
         DO 200 M2=1,MMAX
          DO 150 N2=1,NMAX
          P=JMAX*(I2-1)+J2
          Q=NMAX*(M2-1)+N2
          KQ=(KM(M2)**2+KN(N2)**2)**0.5
          IF (X(I2) .LE. F1 .AND. Y(J2) .LE. F2) THEN
             S3=SINH(KQ*H3)
             B(P,Q)=S3*COS(KM(M2)*X(I2))*COS(KN(N2)*Y(J2))
             W1(P)=-1.0
C
          ELSE
             K1=COSH(KQ*H1)
             K2=COSH(KQ*H2)
             K3=COSH(KQ*H3)
             S1=SINH(KQ*H1)
             S2=SINH(KQ*H2)
             S3=SINH(KQ*H3)
             S13=SINH(KQ*(H1+H3))
             G1=ER**2*S1*S2*S3
             G2=K1*S2*K3+ER*K2*S13+G1
             G3=COS(KM(M2)*X(I2))*COS(KN(N2)*Y(J2))
             G4=K1*S2+ER*K2*S1
             B(P,Q)=KQ*G2*G3/G4
             W1(P)=0.0
             ENDIF
C            WRITE(2,130) P,Q,B(P,Q),W1(P)
C 130        FORMAT(I3,1X,I3,1X,F20.15,1X,F20.15)
C**********
  150     CONTINUE
  200    CONTINUE
  250   CONTINUE
```

```
  300 CONTINUE
C     **Solve the equation (49) in order to compute the Dq**
      DO 655 I3=1,V-1
       B1=ABS(B(I3,I3))
       B1=ABS(B1)
       L=I3
       I4=I3+1
       DO 350 J3 = I4,V
        IF (ABS(B(J3,I3)) .LT. B1) THEN
         GO TO 350
        ELSEIF (ABS(B(J3,I3)) .GE. B1) THEN
         B1=ABS(B(J3,I3))
         L=J3
        ENDIF
C
  350  CONTINUE
       IF (B1 .EQ. 0.0) THEN
         GO TO 850
       ELSEIF (L .EQ. I3) THEN
         GO TO 500
       ELSEIF (L .NE. I3) THEN
         GO TO 400
  400  ENDIF
       DO 450 J4=1,V
        HH1=B(L,J4)
        B(L,J4)=B(I3,J4)
        B(I3,J4)=HH1
  450  CONTINUE
       HH1=W1(L)
       W1(L)=W1(I3)
       W1(I3)=HH1
  500  DO 650 J5=I4,V
        T=B(J5,I3)/B(I3,I3)
        DO 600 K=I4,V
         B(J5,K)=B(J5,K)-T*B(I3,K)
  600   CONTINUE
        W1(J5)=W1(J5)-T*W1(I3)
  650  CONTINUE
  655 CONTINUE
       IF (B(V,V) .EQ. 0.0) THEN
        GO TO 850
       ELSEIF (B(V,V) .NE. 0.0) THEN
        GO TO 660
  660  ENDIF
       D(V)=W1(V)/B(V,V)
       DO 680 I5=(V-1),1,-1
        S6=0.0
        DO 670 J6=I5+1,V
         S6=S6+B(I5,J6)*D(J6)
  670   CONTINUE
       D(I5)=(W1(I5)-S6)/B(I5,I5)
C**********
C      WRITE(2,675) D(I5),I5,D(V)
C 675  FORMAT('D=',F25.18,2X,I5,F25.18)
C**********
```

```
  680   CONTINUE
C
C       **Calculation of Electric Capacitance using Eq.(57)**
        CAPE=0.0
        DO 750 M3=1,MMAX
         DO 700 N3=1,NMAX
          N4=2*N3-1
          Q1=MMAX*(M3-1)+N3
          KQ=(KM(M3)**2+KN(N4)**2)**0.5
          K1=COSH(KQ*H1)
          K2=COSH(KQ*H2)
          K3=COSH(KQ*H3)
          S1=SINH(KQ*H1)
          S2=SINH(KQ*H2)
          S3=SINH(KQ*H3)
          S13=SINH(KQ*(H1+H3))
          C1E=K1*S2*K3+ER*K2*S13+(ER**2)*S1*S2*S3
          C2E=D(Q1)*KQ/(KM(M3)*KN(N4))
          C3E=K1*S2+ER*K2*S1
          C4E=SIN(KM(M3)*A/2)*SIN(KN(N4)*D1/2)
          NCAPE=C1E*C2E*C4E/C3E
          CAPE=CAPE+NCAPE
C         WRITE(2,690) CAPE,NCAPE
C 690     FORMAT(2(1X,F30.25))
          CAPE=CAPE/(PI*9*10**9)
  700    CONTINUE
  750   CONTINUE
       WRITE(2,910)  CAPE
  910 FORMAT('ELECTRIC.CAPACITANCE=',F40.35)
C
C       **Calculation of Magnetic Capacitance using Eq.(57)**
        CAPM=0.0
        DO 800 M5=1,MMAX
         DO 770 N6=1,NMAX
          Q2=MMAX*(M5-1)+N6
          N5=2*N6
          KQ=(KM(M5)**2+KN(N5)**2)**0.5
          K1=COSH(KQ*H1)
          K2=COSH(KQ*H2)
          K3=COSH(KQ*H3)
          S1=SINH(KQ*H1)
          S2=SINH(KQ*H2)
          S3=SINH(KQ*H3)
          S13=SINH(KQ*(H1+H3))
          C1M=K1*S2*K3+ER*K2*S13+(ER**2)*S1*S2*S3
          C2M=D(Q2)*KQ/(KM(M5)*KN(N5))
          C3M=K1*S2+ER*K2*S1
          C4M=SIN(KM(M5)*A/2)*SIN(KN(N5)*D1/2)
          NCAPM=C1M*C2M*C4M/C3M
          CAPM=CAPM+NCAPM
          CAPM=CAPM/(PI*9*10**9)
C         WRITE(2,765) CAPM,NCAPM
C 765     FORMAT(2(1X,F30.25))
  770    CONTINUE
  800   CONTINUE
        WRITE(2,810) CAPM
```

```
  810  FORMAT('MAGNETIC.CAPACITANCE=',F40.35)
C      **Calculation of Gap Capacitance**
       CGAP=(CAPE-CAPM)/4
C      **Calculation of Paracitic Capacitance**
       CPAR=(CAPM-(D1-G)*C0)/2
       WRITE(2,820) CGAP
  820  FORMAT('GAP',1X,'CAP=',F40.35)
       WRITE(2,830) CPAR
  830  FORMAT('PAR.CAP=',F40.35)
C
  850 WRITE(2,900)
  900 FORMAT(1X,'MATRIX CINGULAR NO SOLUTION')
      STOP
      END
```

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